MAE 210C – FLUID MECHANICS III – SPRING 2017

HOMEWORK ASSIGNMENT # 1 (Due at 9:50AM on Monday April 17, 2017)

Problem 1: Consider the model equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -u[u^2 - 2u - (R - R_c)]$$

Obtain the steady solutions $u = U$ and draw the bifurcation diagram. Identify the different bifurcations and investigate the stability of the different branches, both by analyzing the sign of $\mathrm{d}u/\mathrm{d}t$ in the different regions of the bifurcation diagram and by performing a "normal mode" analysis with $u = U + \hat{u}e^{st}$.

Problem 2: (From Drazin, Nonlinear Systems, 1992) A particle is constrained to move around a smooth circular wire of radius a fixed in a vertical plane. Gravity acts on the particle and the plane rotates with constant angular velocity Ω. Show that the particle motion is described by the two first-order differential equations

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega \quad \text{and} \quad a\frac{\mathrm{d}\omega}{\mathrm{d}t} = -g\sin\theta + a\Omega^2\cos\theta\sin\theta,$$

where θ is the angle between the downward vertical and the radius through the particle, as indicated in the figure. Write the problem in dimensionless form. Find the number of positions of equilibrium $\theta = \Theta$ and sketch the bifurcation diagram in the plane $\theta - \Lambda$ with $\Lambda = a\Omega^2/g$. Analyze the stability of the different branches by performing a "normal mode" analysis with $\theta = \Theta + \hat{\theta}e^{st}$ and $\omega = \hat{\omega}e^{st}$.