Problem 1: Consider the model equation
\[
\frac{du}{dt} = -u[u^2 - 2u - (R - R_c)]
\]
Obtain the steady solutions \( u = U \) and draw the bifurcation diagram. Identify the different bifurcations and investigate the stability of the different branches, both by analyzing the sign of \( \frac{du}{dt} \) in the different regions of the bifurcation diagram and by performing a "normal mode" analysis with \( u = U + \hat{u}e^{st} \).

Problem 2: (From Drazin, Nonlinear Systems, 1992) A particle is constrained to move around a smooth circular wire of radius \( a \) fixed in a vertical plane. Gravity acts on the particle and the plane rotates with constant angular velocity \( \Omega \). Show that the particle motion is described by the two first-order differential equations
\[
\frac{d\theta}{dt} = \omega \quad \text{and} \quad a \frac{d\omega}{dt} = -g \sin \theta + a\Omega^2 \cos \theta \sin \theta,
\]
where \( \theta \) is the angle between the downward vertical and the radius through the particle, as indicated in the figure. Write the problem in dimensionless form. Find the number of positions of equilibrium \( \theta = \Theta \) and sketch the bifurcation diagram in the plane \( \theta - \Lambda \) with \( \Lambda = a\Omega^2 / g \). Analyze the stability of the different branches by performing a "normal mode" analysis with \( \theta = \Theta + \hat{\theta}e^{st} \) and \( \omega = \hat{\omega}e^{st} \).