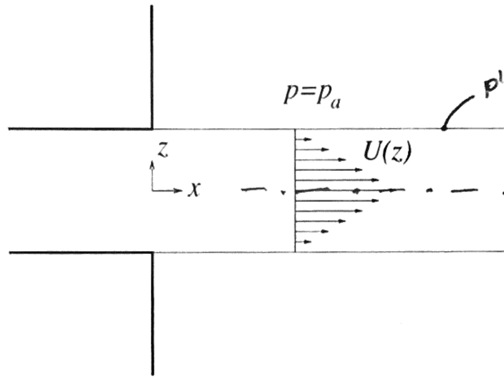
The triangular velocity profile defined by

$$U = 1 - |z| \text{ for } |z| \leq 1 \quad \text{and} \quad U = 0 \text{ for } |z| > 1$$

is proposed as an approximate representation of the velocity across a **liquid sheet discharging into air**. In order to investigate the temporal **inviscid** stability of this flow, integrate Rayleigh's equation to obtain explicit expressions for the phase velocity ω_r/k and growth rate ω_i. Consider separately the sinuous and varicose modes.



$p' = 0 \Rightarrow (U-c)\frac{d\hat{V}_z}{dz} - \frac{dU}{dz}\hat{V}_z = 0$

$\hat{V}_z = A_1 e^{kz} + B_1 e^{-kz}$

$\hat{V}_z = A_2 e^{kz} + B_2 e^{-kz}$

VARICOSE: $A_1 = -B_1$, $\quad -ck(e^k + e^{-k}) + e^k - e^{-k} = 0 \Rightarrow c = \frac{\tanh(k)}{k} = \frac{\omega_r}{k}$, $\omega_i = 0$

ALWAYS NEUTRALLY STABLE

SINUOUS: $B_2 = A_1$, $A_2 = B_1$ $\quad -ck(A_1 e^k - B_1 e^{-k}) + A_1 e^k + B_1 e^{-k} = 0$ ($p' = 0$ AT $z = 1$)

$k(1-c)(A_1 - B_1) + A_1 + B_1 = k(1-c)(B_1 - A_1) - (A_1 + B_1)$

$\omega^2 - k\omega + k\coth(k) - 1 = 0$, $\quad \omega_r + i\omega_i = \frac{k}{2} \pm \sqrt{\frac{k^2 - 4[k\coth(k) - 1]}{4}} = \frac{k}{2} \pm \sqrt{\left(\frac{k}{2}\right)^2 - [k\coth(k) - 1]}$

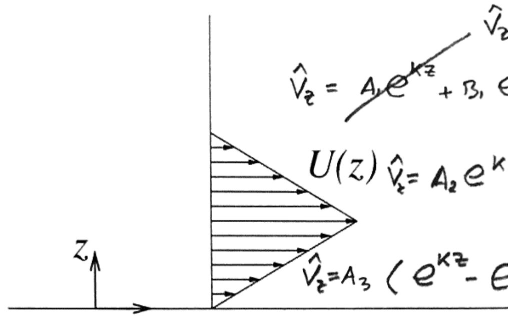
$k > 2.4 \rightarrow$ NEUTRALLY STABLE

$0 < k < 2.4 \rightarrow$ UNSTABLE

The velocity across a wall jet can be approximated by

$$U = z \text{ for } 0 \le z \le 1, \quad U = 2 - z \text{ for } 1 \le z \le 2, \quad \text{and} \quad U = 0 \text{ for } z > 2.$$

Use Rayleigh's equation to analyze the temporal **inviscid** stability of this flow.



$\hat{V}_z(\infty) \to 0$

$\hat{V}_z = \cancel{A_1 e^{kz}} + B_1 e^{-kz}$

$\hat{V}_z = A_2 e^{kz} + B_2 e^{-kz}$

$\hat{V}_z = A_3 (e^{kz} - e^{-kz}) \qquad (\hat{V}_z(0) = 0)$

$$\begin{cases} B_1 e^{-2k} = A_2 e^{+2k} + B_2 e^{-2k} \\ +kcB_1 e^{-2k} = -kc(A_2 e^{2k} - B_2 e^{-2k}) + A_2 e^{2k} + B_2 e^{-2k} \end{cases}$$

$$\begin{cases} A_2 e^{k} + B_2 e^{-k} = A_3 (e^{k} - e^{-k}) \\ k(1-c)(A_2 e^{k} - B_2 e^{-k}) + A_2 e^{k} + B_2 e^{-k} = k(1-c) A_3 (e^{k} + e^{-k}) - A_3 (e^{k} - e^{-k}) \end{cases}$$

$$\omega(A_2 e^{2k} + \cancel{B_2 e^{-2k}}) = A_2 e^{2k}(1-\omega) + B_2 e^{-2k}(1+\cancel{\omega}) \rightarrow \boxed{B_2 = A_2 e^{4k}(2\omega - 1)}$$

$$(k-\omega+2)(e^{2k}-1)A_2 - (k-\omega-2)(1-e^{-2k})B_2 = (k-\omega)(e^{2k}+1)A_2 + (k-\omega)(1+e^{-2k})B_2$$

COMBINING BOTH EXPRESSIONS YIELDS

$$e^{4k}(2\omega-1)\left[e^{-2k} + k - \omega - 1\right] + k - \omega + 1 - e^{2k} = 0$$

WHICH CAN BE SOLVED TO GIVE:

$$\boxed{\omega = -\frac{\left[1-2k-2e^{-2k}+e^{-4k}\right]}{4} \pm \frac{1}{4}\sqrt{(1-2k-2e^{-2k}+e^{-4k})^2 + 8(1-k-2e^{-2k}+(1+k)e^{-4k})}}$$

$\omega_I > 0$ FOR $0.6683 < k < 1.7136$