MAE 210C – FLUID MECHANICS III – SPRING 2017

HOMEWORK ASSIGNMENT # 5 (Due at 11:00AM on Friday June 9, 2017)

Problem The two-dimensional motion and stability of a fluid confined between two long horizontal plates, heated and salted from below, was studied in HWK3 following Huppert & Moore JFM, 78:821–854 (1976). It was seen that introduction of normal modes of the form $(v_z', \theta', \phi') = [\hat{v}_z(z), \hat{\theta}(z), \hat{\phi}(z)]e^{st+i(kx+ly)}$ leads to the problem

$$\begin{aligned}\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \frac{s}{Pr} - \tilde{k}^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \tilde{k}^2\right)\hat{v}_z &= \tilde{k}^2(\hat{\theta} - \gamma\hat{\phi}) \\ \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - s - \tilde{k}^2\right)\hat{\theta} &= -R\hat{v}_z \\ \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - Ls - \tilde{k}^2\right)\hat{\phi} &= -LR\hat{v}_z\end{aligned}$$

to be integrated with boundary conditions

$$z = 0, 1: \qquad \hat{\theta} = \hat{\phi} = \hat{v}_z = 0 \quad \text{and} \quad \frac{\mathrm{d}\hat{v}_z}{\mathrm{d}z} = 0 \text{ (rigid)} \quad \text{or} \quad \frac{\mathrm{d}^2\hat{v}_z}{\mathrm{d}z^2} = 0 \text{ (free)},$$

where $\tilde{k}^2 = k^2 + l^2$ is the total wavenumber. The present homework is concerned with the numerical solution of this problem with free-free boundary conditions. To that end

1. Rearrange the stability equations as an eigenvalue problem for $s = s_r + \mathrm{i}s_i$.
2. For fixed values of $Pr = 1$ and $L = 2$ and the following combinations of R and γ:

$$\begin{aligned}R = 1000: &\quad \gamma = 0.15,\ 0.25 \\ R = 5000: &\quad \gamma = 0.40,\ 0.80,\ 1.10\end{aligned}$$

   compute the associated eigenvalue spectra for $0 \le \tilde{k} \le 4$. Identify the leading eigenvalue(s), i.e., those with maximum real part, and plot their real and imaginary parts as a function of $\tilde{k}$. For each case, determine the wavenumber for which the growth rate s_r is maximum and plot the corresponding spectrum. In view of your results, place each of the five cases on the transition diagram $R(\gamma)$ (see expressions for $R_1(\gamma)$ and $R_2(\gamma)$ in HWK3 solution).
3. Obtain the critical value of γ for $R = 1000$ and $R = 5000$ and indicate the associated wavenumber. Plot the eigenfunctions $[\hat{v}_z(z), \hat{\theta}(z), \hat{\phi}(z)]$ for these two marginal cases.

Note: Please provide a hardcopy of your source code together with your solution.