Problem 1: The steady solution for flow around two-dimensional blunt bodies displays a moderately slender recirculating wake region where we find back flow near the centerline, as shown in the figure.

To investigate the stability characteristics of this flow, we introduce the approximate piecewise linear representation

\[ U = \begin{cases} 
1 & \text{for } |z| \geq 1 \\
1 - S(1 - |z|) & \text{for } |z| \leq 1 
\end{cases} \]

for the base velocity profile, which is scaled with the free-stream velocity, so that \( S \) represents the dimensionless velocity deficit at the centerline (i.e., for \( S < 1 \) we find positive velocity all across the wake, whereas for \( S > 1 \) the velocity is negative near the symmetry plane).

- Use Rayleigh’s equation as a starting point to determine the dispersion relations \( D^s(k, \omega; S) = 0 \) and \( D^v(k, \omega; S) = 0 \) corresponding to sinuous and varicose inviscid perturbations.

- Analyze the temporal stability of the flow using the dispersion relations derived above. Write explicit expressions for the real and imaginary components of the frequency \((\omega_r(k) \text{ and } \omega_i(k))\). Show, in particular, that the varicose mode is always neutrally stable, because \( \omega_i^v = 0 \) for all \( k \) regardless of the value of \( S \), and that the growth rate of the sinuous mode is linearly proportional to \( S \), so that you can write \( (\omega_i)^s/S = f(k) \). Give the function \( f(k) \).

- Analyze the spatial stability of the sinuous mode using the dispersion relation \( D^s(k, \omega; S) = 0 \) derived above. Consider the limit \( S \ll 1 \) and derive approximate expressions for \( k_i(\omega, S) \) and \( k_r(\omega, S) \). Discuss the results in connection with Gaster relation. Consider now increasing values of \( S \). Obtain numerically the solution. Plot the variation of \(-k_i/S \) and \( k_r \) with \( \omega \) for different values of \( S \). Discuss your observations. In particular, identify the value of \( S \) at which the convective/absolute transition takes place for this model velocity profile.