P1: The ejector shown in the figure is used to discharge a container filled with a perfect liquid of density $\rho$ and constant specific heat $c$ into the atmosphere, also at pressure $p_a$. A high-velocity jet of the same liquid, with velocity $U_j$ and cross section $A_j$, is injected along the axis of the discharge pipe, whose cross section is $A$. The jet entrains fluid as it develops, producing an underpressure at the injection section and drawing the fluid from the container. Mixing of the jet with the surrounding liquid is completed downstream, so that at the outlet the liquid discharges as a uniform jet with velocity $U_o$. We want to compute the characteristics of the flow in terms of the injection velocity $U_j$ for given values of $A_j$ and $A$. In the computation, assume that the effect of gravity is negligible.

Address in order the following questions:

1. Assuming that the acceleration of the fluid upstream from the injector is steady and inviscid, write down the equation that relates the ambient pressure $p_a$ with the velocity and pressure at the entrance of the pipe, $U_e$ and $p_e$.

2. Apply the continuity and momentum balance equations to the control volume indicated in the figure. Simplify the result by assuming that the effect of friction on the wall surface is negligibly small.

3. Use the previous equations to show that the volumetric discharge flow rate is given by

$$Q = U_e(A - A_j) = \frac{U_j(A - A_j)}{1 + A/\sqrt{2}A_j(A - A_j)}$$

and that the velocity at the outlet satisfies

$$\frac{U_o}{U_j} = \frac{A_j + 2A_j(A - A_j)}{A + \sqrt{2}A_j(A - A_j)}.$$

4. Assuming that the values of the temperatures $T_e$ and $T_j$ at the entrance section are equal to the ambient value $T_a$, obtain the temperature $T_o$ at the outlet section when the pipe walls are adiabatic.
P2: A liquid of density $\rho$ and specific heat $c$ flows through the elbow-shaped pipe shown below, discharging to the atmosphere as a vertical jet through an outlet section of surface area $A_o$, different from the value $A_i$ at the inlet. Gravity is assumed to have a negligible effect on the fluid motion. Besides, uniform flow properties are assumed at the inlet and outlet sections, where readings from a thermometer give different values $T_i$ and $T_o$ for the temperature. The velocity at the outlet section is measured to be $v_o$ and a manometer is used to determine the value $p_i$ for the pressure at the inlet section. In terms of all of these quantities, determine:

1. The value of the velocity at the inlet $v_i$
2. The two components of the force ($F_x, F_z$) exerted on the pipe.
3. The amount of heat $Q$ transferred per unit time to the liquid by heat conduction through the pipe walls.

P3: A linear array of parallel hot cylinders of radius $R$, whose centers are separated a distance $3R$, is placed perpendicular to a gas stream with velocity $U_1$, density $\rho_1$, and pressure $p_1$, as indicated in the figure. Sufficiently far downstream the gas properties recover uniform values, including a velocity $U_2$ and a pressure $p_2$. Assume that the effect of gravity is negligible. In terms of the quantities indicated in the figure, determine:

1. The value of the density downstream from the array of cylinders.
2. The force acting on each cylinder (per unit spanwise length).
3. The rate of heat transfer from each cylinder to the gas stream (per unit spanwise length).

Express the result with use made of the ratio of specific heats $\gamma$.

NOTE: Explain clearly the simplifications and assumptions employed in the derivation.