Inviscid Counterflow Jets from Aligned Nozzles

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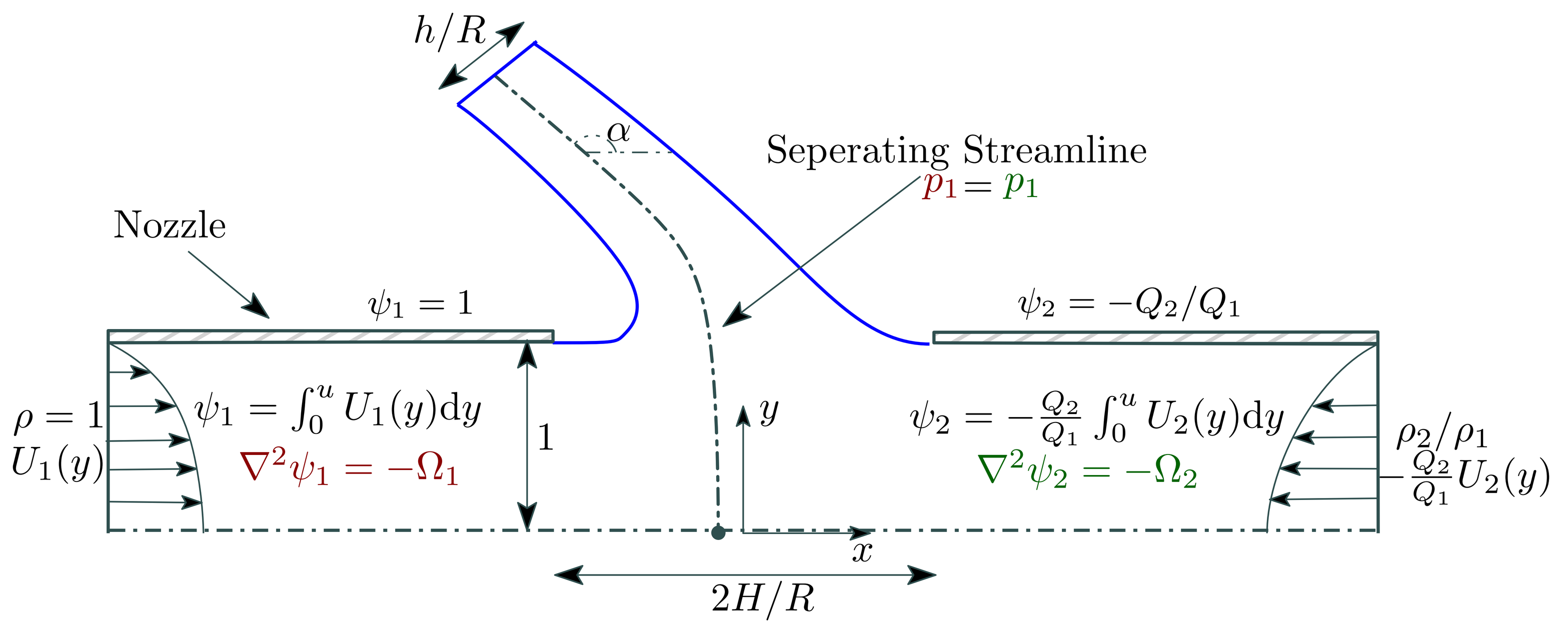
The laminar flow resulting from the impingement of two gaseous jets issuing from aligned planar nozzles of semi-width R separated a distance $2H$ is investigated by numerical and analytical methods, with specific consideration given to the high-Reynolds and low-Mach number conditions typically present in counterflow-flame experiments. The resulting flow, nearly inviscid and effectively incompressible, can be described in terms of a density-weighted stream-function/vorticity formulation that removes the need to consider specifically the boundary separating the two jet streams. Besides the geometric parameter H/R, the solution depends only on the shape of the velocity profiles in the feed-streams and on the jet momentum-flux ratio Λ. While conformal mapping can be used to determine the potential solution corresponding to uniform velocity profiles, numerical integration is required in general to compute vortical flows, including those arising with Poiseuille velocity profiles. Simplified solutions have been found for $H/R \ll 1$ (vertical and potential), and potential solutions for $H/R \sim \mathcal{O}(1)$ and $H/R \gg 1$.

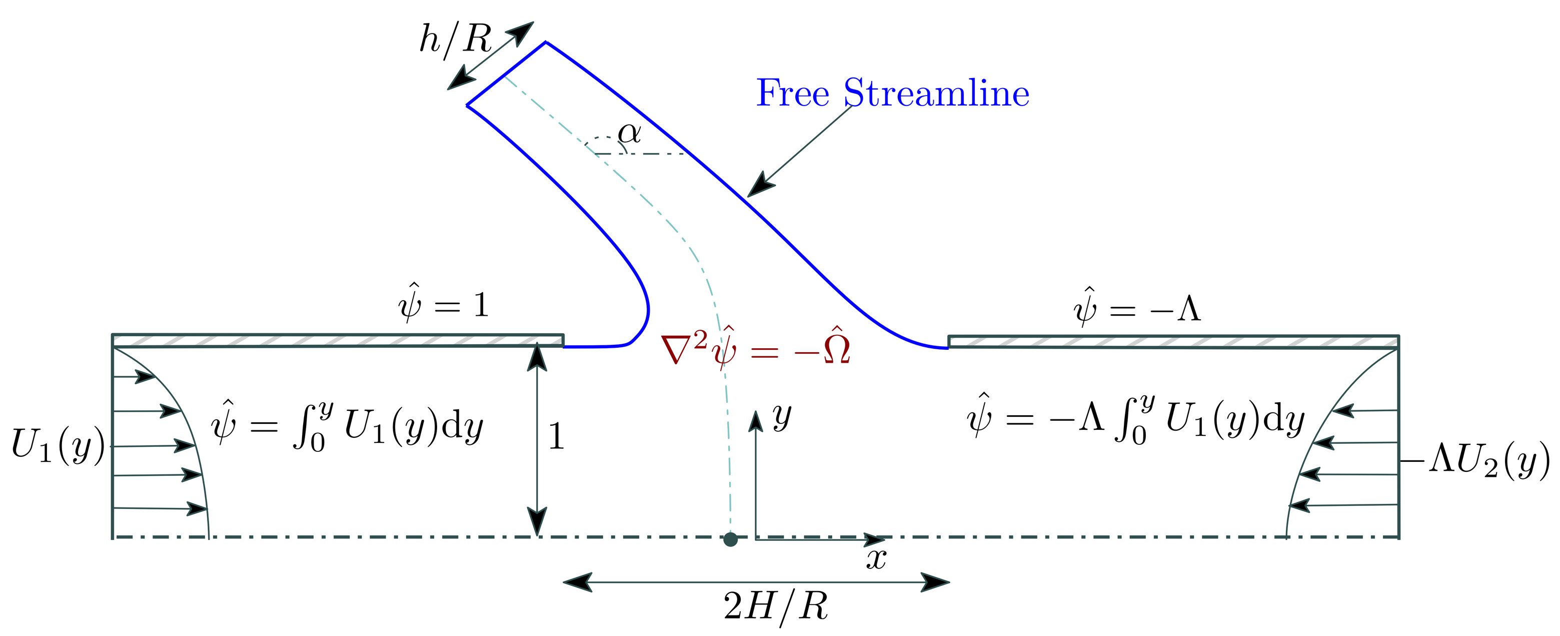
Formulation

The Standard Formulation



- The standard formulation must account for **two** gases and the streamline that separates them.

The Density- Weighted Reduced Formulation



- Introduction of

$$\hat\psi = \begin{cases}\psi, \\ (\rho_2/\rho_1)^{1/2}\psi,\end{cases} \quad \text{and} \quad \hat\Omega = \begin{cases}\Omega, & \text{for} \quad \psi > 0 \\ (\rho_2/\rho_1)^{1/2}\Omega, & \text{for} \quad \psi < 0\end{cases}$$

has reduced the problem to that of a single fluid and eliminated the need to consider the separating streamline.

The Limit $H/R \ll 1$

For the description of the flow inside the nozzles, the opening appears as a point sink of strength $(1+\Lambda)$. The solution can be facilitated by expressing $\hat\psi$ as the sum of a potential stream function $\hat\psi_p$

$$\hat\psi_p = y - \frac{1+\Lambda}{\pi}\arctan\left[\frac{e^{\pi x}\sin(\pi y)}{e^{\pi x}\cos(\pi y)+1}\right], \tag{1}$$

carrying the volume flux of the two streams and a vortical stream function $\hat\psi_v$ with associated zero volume flux.

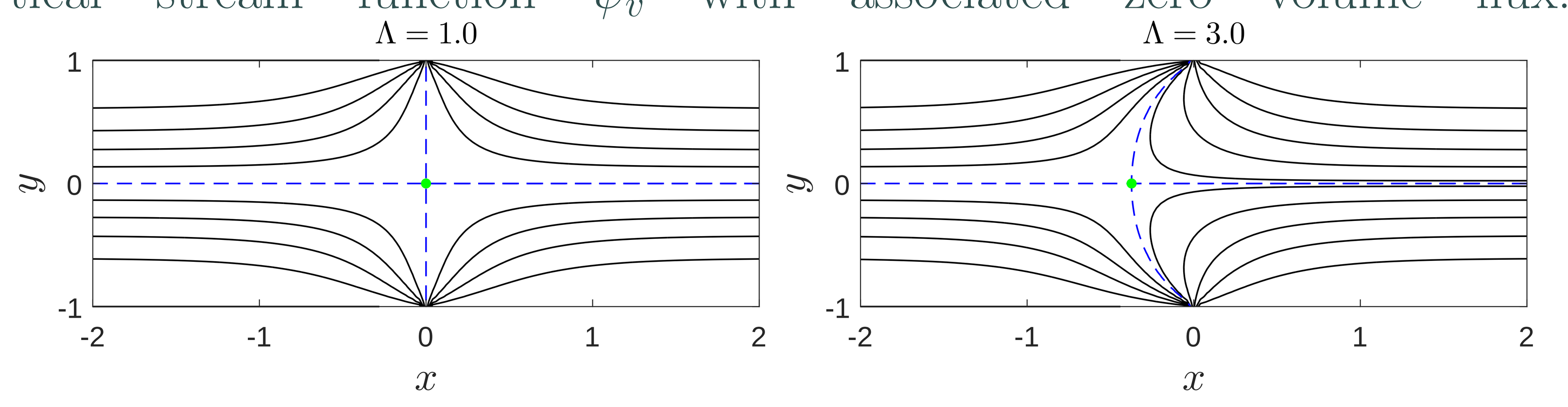


Figure 1: Contours of $\hat\psi$ for Poiseuille distribution in feed-streams with (a) symmetric case $\Lambda = 1$, and (b) $\Lambda = 3$. Blue lines correspond to seperating streamline. The stagnation point is shown in green.

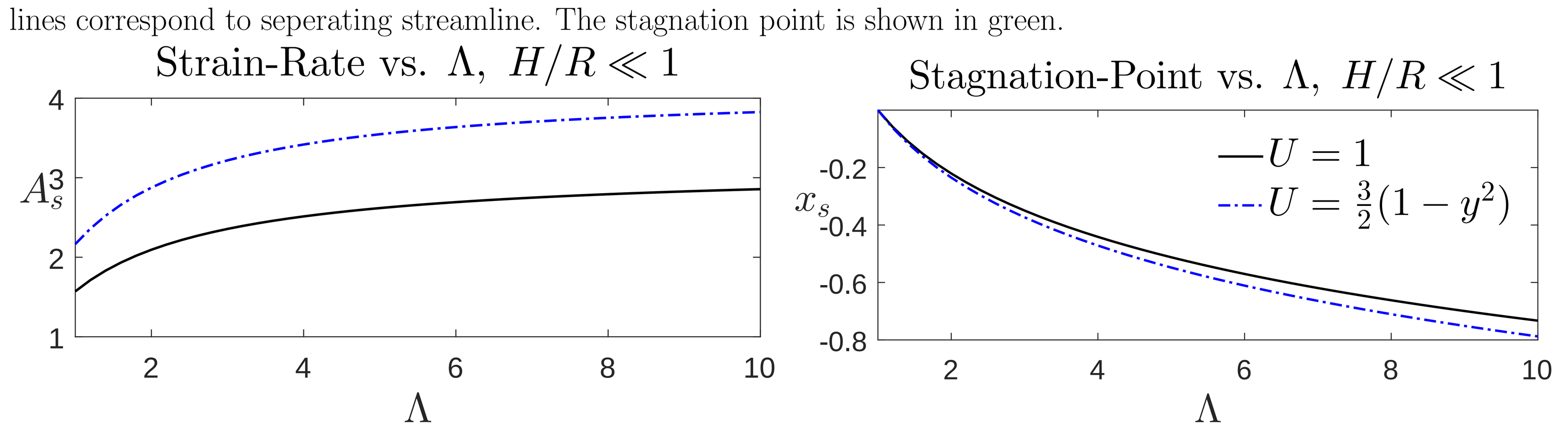


Figure 2: Strain-Rate and stagnation point as functions of momentum-flux ratio Λ for (black) uniform and (blue-dotted) Poiseuille distributions in feed-streams

Potential Flow Solution

For uniform velocity distributions in each feed-stream the entire flow field is **irrotational** and may be described by defining a **complex potential** $w = \hat\phi + \mathrm{i}\hat\psi$ The resulting free jet reaches a uniform velocity profile of magnitude V, direction α, and width h/R, all of which were determined as part of the solution (Figure 4). An axillary function defined by $Q = \log(V/\nu)$ is introduced where $\nu = \mathrm{d}w/\mathrm{d}z$ is the complex velocity. Using Schwarz-Christoffel, w, Q, and ν are each mapped to the upper half ζ-plane via:

$$Q(\zeta) = \cosh^{-1}\zeta = \log(\zeta^2 + \sqrt{\zeta^2 - 1}), \tag{2}$$

$$w(\zeta) = K\left\{\xi\log(\zeta - a_2) + \beta\log(\zeta + a_1) - \gamma\log(\zeta + c)\right\}, \tag{3}$$

$$\nu(\zeta) = \frac{V}{\zeta + \sqrt{\zeta^2 - 1}}. \tag{4}$$

These mappings allow one to write

$$\mathrm{d}z = \frac{K}{V}\frac{\zeta + \sqrt{\zeta^2-1}}{(\zeta + a_1)(\zeta + c)(\zeta - a_2)}\mathrm{d}\zeta. \tag{5}$$

which provides the means for calculating the location of the **free streamlines**, the **separating streamline**, and the **stagnation point** by integrating (5) along appropriate contours in the ζ-plane. The **strain-rate** is also calculated in the ζ-plane. **The case H/R $\gg$ 1** may be treated by setting $V = \Lambda$, corresponding to a jet impinging on a nozzle.

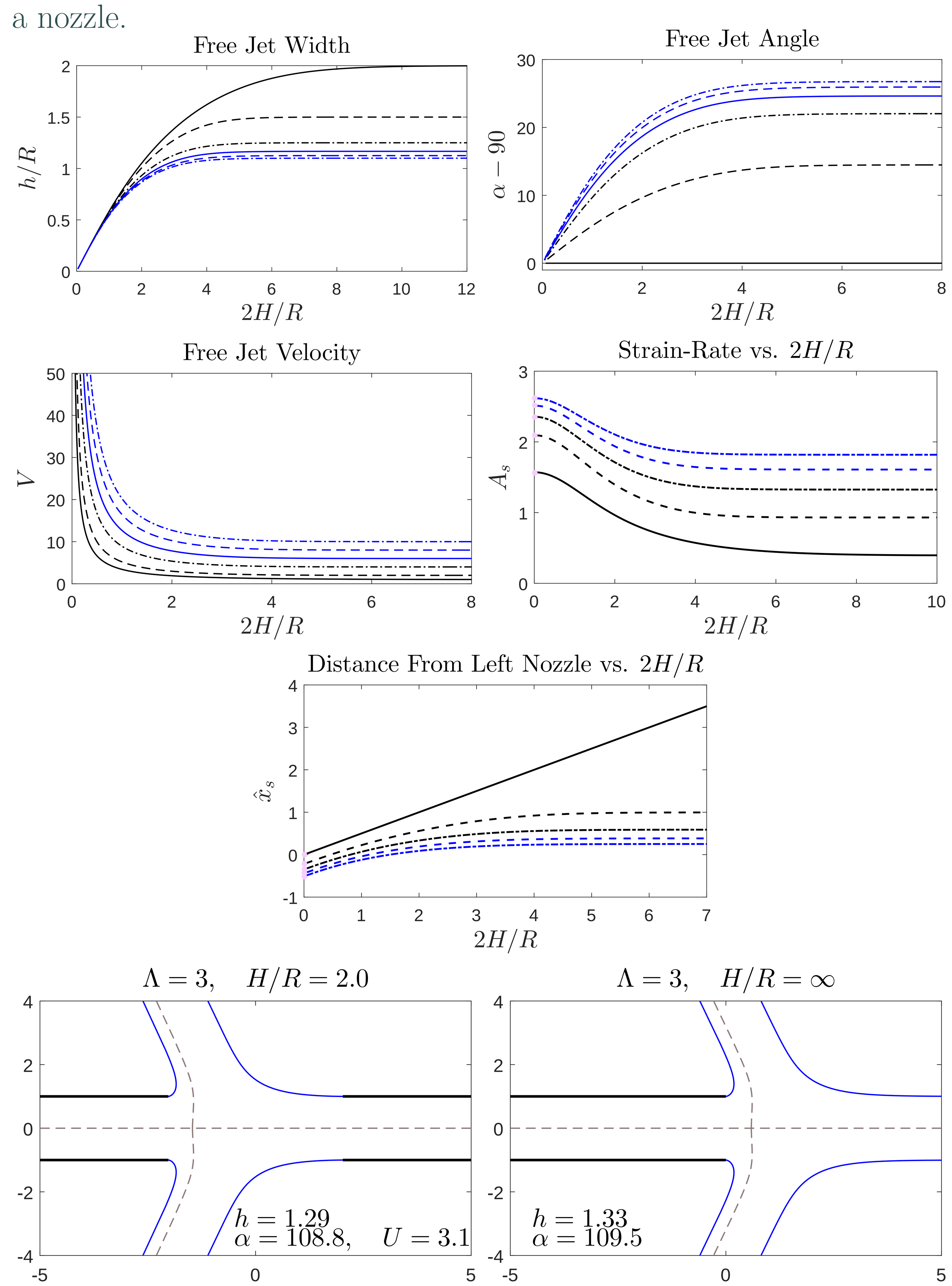


Figure 3: (Upper Rows) Free jet angle α, speed V, width h/R, strain-rate A_s, and distance from nozzle $\hat x$ as a function of nozzle spacing $2H/R$ for various values of momentum-flux ratio Λ. Solid black line corresponds to $\Lambda = 1$. Other curves correspond to increasing values of Λ. (Lower Row) Free and seperating streamlines for $\Lambda = 3$, $H/R = 2$ and $H/R = \infty$ respectively.

Conclusions and ongoing work

Solutions for $H/R \ll 1$ (vortical and potential), $H/R \simeq \mathcal{O}(1)$ & $H/R \gg 1$ (potential) have been described. Extending the solution for vortical flows with H/R order unity is needed.