Problem 1: Use the definition of the entropy $Tds = de + pd(1/\rho)$ to show that, for a perfect gas, the variations of entropy, temperature, density, and pressure are related according to

$$s - s_{\text{ref}} = \frac{c_v}{\gamma} \ln \left( \frac{T}{T_{\text{ref}}} \right) = \ln \left( \frac{p}{p_{\text{ref}}} \right) = \frac{\gamma - 1}{\gamma} \ln \left( \frac{p}{p_{\text{ref}}} \right)$$

Problem 2: Starting from the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0$$

and the inviscid form of the energy equation

$$\rho \frac{De}{Dt} = -p \nabla \cdot \bar{v},$$

derive a conservation equation for the enthalpy $h = e + p/\rho$.

Problem 3: Show that the thrust of a rocket engine with $p_e = p_a$ can be expressed in the form

$$T = \frac{2\gamma}{\gamma - 1} p_e A_e \left[ \left( \frac{p_0}{p_a} \right)^{(\gamma - 1)/\gamma} - 1 \right],$$

where $A_e$ is the transverse area of the nozzle at the exit section and $p_0$ is the stagnation pressure in the nozzle (approximately equal to the pressure in the combustion chamber, where $M \ll 1$).
Problem 4: A Pitot tube, sketched in the figure, is a device used to measure the velocity of a fluid stream. It consists of a slender tube which is placed aligned with the fluid stream, whose values of pressure and velocity are $p_\infty$ and $U_\infty$. Two different pressure readings are taken at two different orifices, internally connected to two different pressure gauges. The static pressure $p_\infty$ is measured through a lateral orifice, where the flow is hardly perturbed. A second orifice is placed at the tube pointed end, where the pressure is equal to the stagnation pressure of the free stream, as follows from the ideal quasisteady deceleration to rest of the flow along the central streamline. Pitots are commonly used in airplanes to measure the flight velocity.

- Show that the flight Mach number can be computed from the two pressure readings according to
  \[ M_\infty = \left\{ \frac{2}{\gamma - 1} \left[ \left( \frac{p_0}{p_\infty} \right)^{(\gamma - 1)/\gamma} - 1 \right] \right\}^{1/2}. \]

- If compressibility is neglected entirely in the calculation, so that Bernoulli’s equation can be used to give $p_0 = p_\infty + \frac{1}{2} \rho_\infty U_\infty^2$ for the pressure at the tube tip, show that the corresponding prediction for the flight Mach number becomes
  \[ M_\infty = \left[ \frac{2}{\gamma} \left( \frac{p_0}{p_\infty} - 1 \right) \right]^{1/2}. \]

- Test the accuracy of this last approximate expression by plotting the resulting prediction for $M_\infty$ as a function of $p_0/p_\infty$ together with the exact value. Comment on the result.