Problem 1: The thrust of a high-altitude vehicle is provided by a solid-propellant rocket engine that discharges to the atmosphere through a convergent-divergent nozzle with exit-to-throat area ratio $A_e/A_t$. When operating in the upper atmosphere, the ambient pressure is so small that in the first approximation we may assume $p_a = 0$.

- Under those conditions, determine the minimum value of $A_e/A_t$ needed to avoid reverse flow near the nozzle rim.

During the reentry, the ambient pressure increases continuously, giving also increasing values of the ratio $p_a/p_0$, where $p_0$ is the constant stagnation pressure in the combustion chamber upstream from the nozzle entrance. For the value of $A_e/A_t$ calculated previously, determine the value of $p_a/p_0$ for which:

- The nozzle discharges to the ambient as a supersonic jet without expansions or shocks.
- There exists a normal shock wave standing at the nozzle exit.
- The nozzle unblocks, so that the flow is subsonic everywhere along the nozzle, except at the throat, where $M = 1$.

Problem 2: An air container at pressure $p_c$ and density $\rho_c$ discharges to an open atmosphere at pressure $p_a < p_c$ through a convergent-divergent nozzle with exit-to-throat area ratio $A_e/A_t = 4$.

1. Obtain the values of $p_a/p_c$ for which (i) the flow is everywhere subsonic, except at the throat, where it is sonic, (ii) the nozzle discharge occurs as a supersonic jet with $p_c = p_a$, and (iii) a normal shock is found at the exit section.

2. For $p_a/p_c = 0.99$, obtain the values of the Mach number at the exit and at the throat $M_e$ and $M_t$. Determine the mass flow rate $\dot{m}$, giving the result in the form $\dot{m}/(\sqrt{\gamma \rho_c p_c A_e})$.

3. For $p_a/p_c = 0.1$, obtain the values of $M_e$ and $M_t$ as well as the mass flow rate $\dot{m}/(\sqrt{\gamma \rho_c p_c A_e})$. An oblique shock is formed at the exit section. Determine the Mach number immediately downstream as well as the deflection angle.

4. For $p_a/p_c = 0.025$, obtain the values of $M_e$ and $M_t$ as well as the mass flow rate $\dot{m}/(\sqrt{\gamma \rho_c p_c A_e})$. The expansion formed at the exit section is a Prandtl-Meyer expansion in the vicinity of the nozzle rim. Determine the Mach number found downstream as well as the deflection angle.