Problem 1: According to the International Standard Atmosphere, the pressure distribution in the lower atmosphere (i.e. for altitudes in the range $0 \leq z \leq 11,000$ m) is given by

$$p(\text{atm}) = \left(1 - \frac{z}{44,332\text{m}}\right)^{5.256}.$$ 

A small military rocket is to be designed to generate $T = 10^4$ N at a cruise altitude $z_c = 5,000$ m. The temperature and pressure in the combustion chamber are $p_0 = 50$ atm and $T_0 = 3,000$ K, respectively. In the calculations assume $\gamma = 1.4$ and $R_g = 287 \text{ J/(kg K)}$. Determine:

1. The specific impulse $I_{sp}$ (in s).
2. The mass flow rate $\dot{m}$ (in kg/s).
3. The characteristic velocity $c^* = p_0 A^*/\dot{m}$ (in m/s).
4. The throat area $A^*$ (in m$^2$).
5. The exit area $A_e$ (in m$^2$).
6. The Mach number at the exit section $M_e$.

If the rocket moves to an altitude of $z = 10,000$ m, determine:

7. The thrust $T_{10}$ (in N).
8. The deflection of the stream through the Prandtl-Meyer expansion that forms at the nozzle rim.

An oblique shock forms outside the nozzle as the rocket descends from cruise altitude towards the earth surface.

9. Obtain the thrust $T_0$ at $z = 0$ (in N).
10. At that altitude, find the deflection across the oblique shock and the Mach number found immediately downstream.

Problem 2: We want to build a small solid-propellant rocket for lower-atmosphere operation ($p_a \simeq 1$ atm) using a nozzle of throat area $A^* = 23.615$ cm$^2$ and exit area $A_e = 100$ cm$^2$. If the working fluid is assumed to be a perfect gas with $\gamma = 1.4$ and $R_g = 287 \text{ J/(kg K)}$, determine the following

- Pressure in the combustion chamber $p_0$ for which the pressure at the nozzle exit matches the ambient value ($p_e = p_a$).
- Thrust $T$.

Assuming a temperature $T_0 = 3000$ K in the combustion chamber, compute

- Specific impulse $I_{sp}$.
- Mass flow rate $\dot{m}$.

While the temperature in the combustion chamber remains approximately constant while the propellant is burning, the mass flow rate changes as the solid propellant surface retreats. To investigate the influence of those changes on the performance of the rocket calculate the thrust and the specific impulse if the mass flow rate were either twice or half of the value computed above.