Problem 1: Two options are being considered for the propulsion device of a long-range missile designed to fly at a cruise Mach number $M = 2.0$ at an altitude where $T_a = 220$ K:

- A Ramjet with peak temperature $T_{04} = 2900$ K
- A Turbojet with compressor pressure ratio $p_{rc} = 20$ and peak temperature $T_{04} = 1800$ K

The nozzle in both designs will have the exit area required to match the ambient pressure at the exit section ($p_e = p_a$). Because weight is a key issue, the option preferred is the one that provides a higher specific thrust $T/\dot{m}_a$. In the calculation, assume that the two engines behave as ideal and that the gas is a perfect gas with $\gamma = 1.4$ and $R = 287$ J/(kg K). In both cases, use $Q_R = 45 \times 10^6$ J/kg for the fuel heat of reaction.

1. Determine the values $T/\dot{m}_a$ for both engines. Based on the result, select the propulsion device for the application at hand.

2. Obtain in both cases the associated thrust specific fuel consumption TSFC.
Problem 2: The figure below represents a schematic view of a **turbofan** with small bypass ratio $B = 3$, similar to the Pratt & Whitney F119 developed for the F22 Raptor, including an afterburner downstream from the turbine. To determine the associated specific thrust $T/\dot{m}_a$ at **take off** assume that the diffuser is ideal, so that $p_0 = p_a$ and $T_0 = T_a$. In the calculations, use $a_a = 340$ m/s for the ambient sound speed and $\gamma = 1.4$ for the ratio of specific heats.

1. For a fan with pressure ratio $p_{rf} = 1.5$ and adiabatic efficiency $\eta_f = 0.85$, obtain the temperature ratio $T_{08}/T_a$.

2. Assuming that the bypass flow is **expanded isentropically** in the fan nozzle to reach the ambient pressure at the exit, obtain the exit Mach number $M_{ef}$ and the associated exit velocity $u_{ef}$.

3. For a compressor with pressure ratio $p_{rc} = 15$ and adiabatic efficiency $\eta_c = 0.85$, obtain the temperature ratio $T_{03}/T_a$.

4. Using the condition $P_{sc} + P_{sf} = P_{st}$, compute the temperature ratio across the turbine $T_{05}/T_{04}$. In the calculation, use $T_{04}/T_a = 5$ for the peak temperature at the turbine inlet.

5. If the turbine adiabatic efficiency is $\eta_t = 0.95$, determine the pressure ratio $p_{05}/p_a$. In the calculation, neglect pressure losses across the combustion chamber.

6. Consider that the combustion process in the afterburner increases the stagnation temperature by a factor of three (i.e., $T_{06} = 3T_{05}$) with a negligible pressure loss (i.e., $p_{06} \approx p_{05}$). If the nozzle is ideal, determine the Mach number at the exit $M_e$ and the associated jet speed $u_e$.

7. Assuming a small fuel-to-air mass-flow ratio $f \ll 1$, calculate the specific thrust $T/\dot{m}_a$ of the turbofan at take off.
Problem 3: The figure below represents a turboshaft that is used on a small airplane flying at \( M = 0.5 \) at an altitude where \( T_a = 260 \text{ K} \). Since the gas is expanded through the power turbine to a pressure close to the ambient value (\( p_6 \approx p_a \)), the resulting exhaust jet has a small speed \( u_e \ll u \) that does not contribute significantly to the thrust. To obtain the specific thrust \( T/\dot{m}_a \) and the TSFC of the turboshaft follow the steps suggested below. In the calculations, use \( \gamma = 1.4 \) and \( R = 287 \text{ J/(kg K)} \).

1. Assuming that the diffuser has an efficiency \( \eta_d = 0.98 \), determine the stagnation values of the temperature and pressure at the compressor inlet, giving the results in the form \( p_{02}/p_a \) and \( T_{02}/T_a \).

2. For a compressor with pressure ratio \( p_{rc} = 15 \) and adiabatic efficiency \( \eta_c = 0.85 \), obtain the temperature ratio \( T_{03}/T_{02} \).

3. Assuming that the maximum temperature at the turbine inlet is \( T_{04} = 1500 \text{ K} \), use the energy balance across the combustion chamber to determine the value of the fuel-to-air ratio \( f \). In the calculation, employ \( Q_R = 45 \times 10^6 \text{ J/kg} \) and \( \eta_b = 1 \).

4. Using the condition \( P_{sc} = P_{st} \), determine the value of the temperature ratio across the compressor turbine \( T_{05}/T_{04} \).

5. If the compressor turbine adiabatic efficiency is \( \eta_t = 0.95 \), determine the pressure ratio \( p_{05}/p_{04} \).

6. Analyze the expansion across the power turbine to determine the associated shaft power \( P_{sp} \), giving the result in the dimensionless form \( P_{sp}/(\dot{m}_au^2) \). In the computation, neglect pressure losses across the combustion chamber (i.e., \( p_{04} = p_{03} \)) and use \( \eta_{pt} = 0.96 \) for the adiabatic efficiency of the power turbine.

7. Taking into account that \( u_e \ll u \), calculate the specific thrust \( T/\dot{m}_a \) and TSFC for the turboshaft. The efficiencies of the propeller and the gear box are \( \eta_{pr} = 0.8 \) and \( \eta_g = 0.93 \), respectively.