The Concorde Rolls-Royce Olympus 593 engines were designed to cruise at \( M = 2 \) at an altitude of 53,000 feet \((T_a = 216\, \text{K})\). At the time, the maximum turbine inlet temperature was limited to \( T_{01} = 1350\, \text{K} \). The engine employed a compressor with pressure ratio \( p_{rc} = 11.3 \). In trying to understand the selection of the latter value it is of interest to determine the variation of \( T / \dot{m}_a \), TSFC, and \( \eta_o \) with \( p_{rc} \) for the Olympus 593 engine, assuming the following values for the efficiencies and combustor pressure ratio: \( \eta_n = 0.97, \eta_d = 0.94, \eta_c = 0.80, \eta_t = 0.88, \eta_b = 1.0, \) and \( r_c = 1.0 \). In the calculation, use \( \gamma = 1.4, R = 287\, \text{J/(kg K)} \), and \( Q_R = 45 \times 10^6\, \text{J/kg} \).
The figure below represents a schematic view of a turbofan with small bypass ratio \( R = 3 \), similar to the Pratt & Whitney F119 developed for the F22 Raptor, including an afterburner downstream from the turbine. To determine the associated specific thrust \( T/\dot{m}_a \) at takeoff, assume that the diffuser is ideal, so that \( p_{0e} = p_a \) and \( T_{0e} = T_a \). In the calculations, use \( a_a = 340 \) m/s for the ambient sound speed and \( \gamma = 1.4 \) for the ratio of specific heats.

1. For a fan with pressure ratio \( p_{f} = 1.5 \) and adiabatic efficiency \( \eta_f = 0.85 \), obtain the temperature ratio \( T_{0f}/T_a \): 
   \[ T_{0f}/T_a = 1 + \frac{p_{f}}{\eta_f} = 1.19 \]

2. Assuming that the bypass flow is expanded isentropically in the fan nozzle to reach the ambient pressure at the exit, obtain the exit Mach number \( M_{e,f} \) and the associated exit velocity \( u_{e,f} \): 
   \[ \frac{T_{0e}^*}{T_a} = \frac{T_{0f}}{T_a} = 1.19 \]

3. For a compressor with pressure ratio \( p_{c} = 15 \) and adiabatic efficiency \( \eta_c = 0.85 \), obtain the temperature ratio \( T_{0c}/T_a \): 
   \[ T_{0c}/T_a = 1 + \frac{p_{c}}{\eta_c} = 2.37 \]

4. Using the condition \( P_{e,1} + P_{e,2} = P_a \), compute the temperature ratio across the turbine 
   \[ T_{0t}/T_{0a} \]. In the calculation, use \( T_{0a}/T_a = 5 \) for the peak temperature at the turbine inlet.
   \[ T_{0s}/T_{0a} = 1 - \frac{\left[ (p_{e,2} + 1)/\eta_c + (p_{e,1} + 1)/\eta_f \right]}{(T_{0a}/T_a)} = 0.6385 \]

5. If the turbine adiabatic efficiency is \( \eta_t = 0.90 \), determine the pressure ratio \( p_{0t}/p_{0a} \). In the calculation, neglect pressure losses across the combustion chamber.
   \[ p_{0t}/p_{0a} = \left[ 1 - \frac{\left( T_{0a}/T_a \right)}{\eta_t} \right] \frac{T_{0s}}{T_{0a}} = 2.655 \]

6. Consider that the combustion process in the afterburner increases the stagnation temperature by a factor of three (i.e., \( T_{0b} = 3T_{0a} \)) with a negligible pressure loss (i.e., \( p_{0b} \approx p_{0a} \)). If the nozzle is ideal, determine the Mach number at the exit \( M_e \) and the associated jet speed \( u_0 \).

7. Assuming a small fuel-to-air mass-flow ratio \( f \ll 1 \), calculate the specific thrust \( T/\dot{m}_a \) of the turbofan at takeoff.

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![Diagram](image.png)

6. \( \frac{P_{0e}}{P_a} = \frac{P_{0f}}{P_{0e}} \) \( \frac{T_{0f}}{T_{0e}} = 3 \)

7. \( \frac{T_{0f}}{T_{0a}} = \sqrt{\frac{T_{0s}}{T_{0a}}} \frac{T_{0f}}{T_{0s}} = 7.12 \)

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7. \( \frac{u_0}{u_{e,f}} = 1.19 \times 3 \times 269 = 199.7 \text{ m/s} \)
The figure below represents a turboshaft that is used on a small airplane flying at \( M = 0.5 \) at an altitude where \( T_a = 260 \text{ K} \). Since the gas is expanded through the power turbine to a pressure close to the ambient value \((p_6 \approx p_a)\), the resulting exhaust jet has a small speed \( u_e \ll u \) that does not contribute significantly to the thrust. To obtain the specific thrust \( T/\dot{m}_a \) and the TSFC of the turboshaft follow the steps suggested below. In the calculations, use \( \gamma = 1.4 \) and \( R = 287 \text{ J/(kg K)} \).

1. Assuming that the diffuser has an efficiency \( \eta_d = 0.98 \), determine the stagnation values of the temperature and pressure at the compressor inlet, giving the results in the form \( p_{01}/p_a \) and \( T_{01}/T_a \).

2. For a compressor with pressure ratio \( p_{rc} = 15 \) and adiabatic efficiency \( \eta_c = 0.85 \), obtain the temperature ratio \( T_{01}/T_{02} \).

3. Assuming that the maximum temperature at the turbine inlet is \( T_{04} = 1500 \text{ K} \), use the energy balance across the combustion chamber to determine the value of the fuel-to-air ratio \( f \). In the calculation, employ \( Q_R = 45 \times 10^6 \text{ J/kg} \) and \( \eta_b = 1 \).

4. Using the condition \( P_{sc} = P_{st} \), determine the value of the temperature ratio across the compressor turbine \( T_{03}/T_{04} \).

5. If the compressor turbine adiabatic efficiency is \( \eta_t = 0.95 \), determine the pressure ratio \( p_{05}/p_{04} \).

6. Analyze the expansion across the power turbine to determine the associated shaft power \( P_{st} \), giving the result in the dimensionless form \( P_{st}/(\dot{m}_a u_e^2) \). In the computation, neglect pressure losses across the combustion chamber (i.e., \( p_{04} = p_{03} \)) and use \( \eta_{pt} = 0.96 \) for the adiabatic efficiency of the power turbine.

7. Taking into account that \( u_e \ll u \), calculate the specific thrust \( T/\dot{m}_a \) and TSFC for the turboshaft. The efficiencies of the propeller and the gear box are \( \eta_{pr} = 0.8 \) and \( \eta_g = 0.93 \), respectively.
1. Find \( f \)

\[
\text{equation sheet:} \quad f = \frac{\left( \frac{T_{av}}{T_a} \right) - \left[ 1 + \left( \frac{P_{st}}{P_{st}} - 1 \right) \right] \left( 1 + \frac{Y_{st} - 2}{M^2} \right)}{\frac{M_b \rho R e}{C_p T_a} - \frac{T_{av}}{T_a}}
\]

\[= \frac{0.0235}{T_{av}}
\]

2. Find \( \frac{T_{st}}{T_a} \)

Using \( P_{st} = P_{sc} \) together with \( \frac{T_{st}}{T_a} = 1 + \frac{Y_{st} - 2}{M^2} \), \( \frac{T_{st}}{T_{av}} = 1 + \frac{P_{st} Y_{st} - 1}{T_{av} Y_{st} - 1} \)

one obtains (or using eqn 20 directly)

\[
\frac{T_{st}}{T_{av}} = 1 - \left( \frac{P_{st} Y_{st} - 1}{T_{av} Y_{st} - 1} \right) \left( 1 + \frac{Y_{st} - 2}{M^2} \right)
\]

\[= 0.8095
\]

Equation (26) should not be applied since it's derived for \( P_{st} = P_{sc} + P_{st} \). In this problem, a separate turbine i.e. the power turbine is powering the fan, such that \( P_{st} = P_{sc} \) whereas \( P_{st} = P_{st} \).

Thus \( \frac{T_{st}}{T_a} = \frac{T_{st}}{T_{av}} \cdot \frac{T_{av}}{T_a} = 5.666 \)

3. Find \( \frac{P_{st}}{P_a} \)

\[
\frac{P_{st}}{P_a} = \frac{P_{st}}{P_{ou}} \cdot \frac{P_{ou}}{P_{sc}} \cdot \frac{P_{sc}}{P_{k}} = \frac{P_{st}}{P_{ou}} \cdot \frac{P_{sc}}{P_{k}} = \frac{P_{st}}{P_{ou}} \cdot \frac{P_{sc}}{P_{k}}
\]

We determine \( P_{st}/P_{ou} \) and \( P_{st}/P_{sc} \) from the adiabatic efficiencies of the c. turbine and diffuser respectively.

\[
\eta_t = \frac{T_{st} - T_{ou}}{T_{st} - T_{ach}} = \frac{T_{st} - T_{ou}}{P_{st}/P_{ou}} = 1 \Rightarrow P_{st}/P_{ou} = \left( 1 + \frac{T_{st}}{T_{ach}} - 1 \right)^{1/11} = 0.3567
\]

\[
\eta_d = \frac{T_{st} - T_{ach}}{T_{st} - T_{ach}} = \frac{(P_{sc}/P_{ach})^{1/11} - 1}{T_{st}/P_{ou} - 1} \Rightarrow P_{st}/P_{ach} = \left( 1 + \frac{Y_{st} - 2}{M^2} \eta_{st} - 1 \right)^{1/11} = 1.4320
\]

\[
\Rightarrow \frac{P_{st}}{P_a} = \left( 1 + \frac{T_{st}}{T_{ach}} - 1 \right)^{1/11} \cdot \frac{P_{st}}{P_{ach}} \cdot \frac{P_{ach}}{P_{k}} \cdot \frac{P_{k}}{P_a} = 9.2
\]
4. Find $T_{o_2}/T_a$ and $P_{o_2}/P_a$

\[
\eta_f = \frac{T_{o_1} - T_a}{T_{o_2} - T_a} = \frac{P_{o_2}/P_a}{T_{o_2}/T_a} - 1 = \frac{P_f}{T_{o_2}/T_a} - 1 \Rightarrow \frac{T_{o_2}}{T_o} = 1 + \frac{P_f}{\eta_f} \frac{\eta_f}{T_{o_2}/T_a} - 1
\]

Therefore:

\[
\frac{T_{o_2}}{T_a} = \frac{T_{o_2}}{T_{o_2}} = \left(1 + \frac{P_f}{\eta_f} \right)^{\frac{1}{1 + \frac{\eta_f}{2} M^2}} = 1.32
\]

\[
\frac{P_{o_2}}{P_a} = \frac{P_{o_2}}{P_{o_2}/P_a} = P_f \left(1 + \eta_f \frac{\eta_f}{2} M^2 \right)^{\frac{1}{1 + \frac{\eta_f}{2} M^2}} = 2.864
\]

5. Find $u_{ef}/a_a$

Equation sheet (25) \Rightarrow \frac{\eta_f}{1 - \frac{\eta_f}{2} \frac{u}{a_a}} = \frac{\eta_f}{1 + \frac{P_f}{\eta_f}} \left(1 + \frac{\eta_f}{2} M^2 \right)^{-1} \left(1 - \frac{P_f}{\eta_f} \frac{\eta_f}{(1 + \frac{\eta_f}{2} M^2)} \right)

Thus \[u_{ef}/a_a = 1.356\]

6. Find $T_{o_2}/T_a$

Using $P_f = P_{o_2}$, one obtains:

\[m_a B C_P (T_{o_2} - T_o) = m_a (1 + f) C_P (T_{o_2} - T_{o_2})\]

or

\[B \left(\frac{T_{o_2}}{T_a} - \frac{T_{o_2}}{T_a}\right) = (1 + f) \left(\frac{T_{o_2}}{T_a} - \frac{T_{o_2}}{T_a}\right)\]

\[\Rightarrow \frac{T_{o_2}}{T_a} = \frac{T_{o_2}}{T_a} + \frac{B \left(\frac{T_{o_2}}{T_a} - \frac{T_{o_2}}{T_a}\right)}{(1 + f)} = 3.92\]

7. Find $P_{o_2}/P_a$

\[
P_{o_2}/P_a = \frac{P_{o_2}}{P_{o_2}/P_a} \text{ where } P_{o_2}/P_{o_5} \text{ determined from adiabatic efficiency of power turbine}:
\]

\[
\eta_{pt} = \frac{T_{o_2} - T_{o_5}}{T_{o_2} - T_{o_5}} = \frac{T_{o_2}/T_{o_5} - 1}{(P_{o_2}/P_{o_5})^{\frac{1}{\eta_{pt}}} - 1} \Rightarrow \frac{P_{o_2}}{P_{o_5}} = \left(1 + \frac{T_{o_2}/T_a}{T_{o_2}/T_{o_5}} - 1\right)^{\frac{1}{1 + \frac{\eta_{pt}}{2} M^2}} = 0.142
\]

Thus \[\frac{P_{o_2}}{P_a} = 1.49\]
8. Find \( \frac{u_e}{a_a} \) \((P_e = P_a)\)

\[ h_{eb} = h_{os_b} = h_{eb} = h_{eb} + u_e^2/2 \]

or

\[ T_{sb} - T_e = \frac{u_e^2}{2C_p} \]

therefore

\[ \gamma_n = \frac{T_{sb} - T_e}{T_{sb} - T_{ef}} = \frac{\frac{u_e^2}{2C_p}}{T_{sb} \left(1 - \left(\frac{P_e}{P_{os_b}}\right)^{\gamma - 1}\right)} \]

Solving for \( \frac{u_e^2}{a_a^2} \)

\[ \frac{u_e^2}{a_a^2} = \frac{2C_p T_{sb}}{a_a^2 T_{Ta}} \left(1 - \left(\frac{P_e}{P_{os_b}}\right)^{\gamma - 1}\right) \gamma_n \]

\[ \frac{C_p T_{Ta}}{a_a^2} = \frac{\gamma M T_{Ta}}{\gamma - 1} \]

thus

\[ \frac{u_e}{a_a} = 1.4943 \]

9. Find \( \frac{T}{m_a} \)

equation (21)

\[ \frac{T}{m_a} = a_a \left[\left(1 + \frac{f}{a_a}\right) \frac{u_e}{a_a} + B \frac{u_e f}{a_a} - (1 + B) M\right] = 1.5542 \text{ kN/s} \text{ kg}^{-1} \]

10. For \( M = 0 \), turboshaft mode, find \( f, \frac{T_{sa}}{T_{Ta}}, \frac{P_{sa}}{P_a} \)

using formulas derived in parts 1, 2, and 3, evaluated with \( M = 0 \) gives

\[ f = 0.0246 \quad \frac{T_{sa}}{T_{Ta}} = 5.785 \quad \frac{P_{sa}}{P_a} = 7.11 \]

11. For \( M = 0 \), turboshaft mode, find \( \frac{P_{pt}}{m_a a_a^2} \) \((P_{os_b} = P_a)\)

\[ \frac{P_{pt}}{m_a a_a^2} = \left(\frac{T_{os_b}}{T_{Ta}} - \frac{T_{sb}}{T_{Ta}}\right) = \left(\frac{T_{os_b}}{T_{Ta}} - 1\right) \left(1 + \frac{f}{a_a^2} \frac{T_{os_b}}{T_{Ta}}\right) \]

therefore

\[ \frac{P_{pt}}{m_a a_a^2} = \frac{1 + f}{\gamma - 1} \left(\frac{T_{os_b}}{T_{Ta}} - \frac{T_{sb}}{T_{Ta}}\right) = 6.4734 \]