Problem 1: Consider a turbojet with compressor pressure ratio $P_r = 20$, nozzle exit area $A_e = 0.5 \text{ m}^2$, peak temperature $T_0 = 1800 \text{ K}$, and adiabatic efficiencies (diffuser, compressor, turbine, and nozzle) $\eta_d = 0.85$, $\eta_c = 0.8$, $\eta_t = 0.9$, and $\eta_n = 1$ (i.e., the nozzle is assumed to be ideal). Follow the steps listed below to calculate the air mass flux

$$\dot{m}_a = \frac{p_e u_e A_e}{1 + f} = \frac{\gamma}{1 + f} \frac{p_a A_e}{a_a} M_e \left( \frac{p_e}{p_a} \right) \left( \frac{T_e}{T_a} \right)^{-1/2}$$

and the thrust

$$T = \dot{m}_a a_a [(1 + f) u_e/a_a - M_e] + A_e (p_e - p_a)$$

both at cruise conditions (36,000 feet) and at takeoff. In the calculations, assume that the effective heat of reaction is $\eta_b Q_R = 45 \times 10^6 \text{ J/kg}$, that the pressure loss in the combustor is negligible (i.e. $r_c = \frac{p_0}{p_0_3} = 1$), and that $\gamma = 1.4$ and $R_g = 287 \text{ J/(kg K)}$, corresponding to $c_p = 1,004.5 \text{ J/(kg K)}$.

During flight at cruise conditions ($M = 1.5$, $p_a = 22,600 \text{ Pa}$, and $T_a = 217 \text{ K}$):

1. Determine the fuel-to-air mass ratio $f$.
2. Obtain the temperature jump across the turbine $T_{0_b}/T_{0_a}$.
3. Assuming that the nozzle is dimensioned to give $p_e = p_a$ at cruise, calculate the exit Mach number $M_e$, along with the exit temperature $T_e/T_a$ and the exit velocity $u_e/a_a$.
4. Compute the nozzle throat-to-exit area ratio $A_t/A_e$.
5. Determine the air mass flux $\dot{m}_a$ (kg/s) and the thrust $T$ (kN).

At takeoff ($M = 0$, $p_a = 101,300 \text{ Pa}$, and $T_a = 288 \text{ K}$):

6. Determine the fuel-to-air mass ratio $f$.
7. Obtain the temperature jump across the turbine $T_{0_b}/T_{0_a}$ and the pressure behind the turbine, giving the latter in the form $p_{0_b}/p_a$.
8. The nozzle has fixed geometry (i.e., the value of $A_t/A_e$ is that computed above). Assuming that the nozzle remains choked at takeoff, with an oblique shock forming right outside the nozzle exit, obtain the values of the Mach number $M_e$, temperature $T_e/T_a$, velocity $u_e/a_a$, and pressure $p_e/p_a$ at the nozzle exit section.
9. Obtain the deflection of the jet stream across the oblique shock near the nozzle rim $\delta$, as well as the corresponding post-shock Mach number $M_d$.
10. Compute the takeoff values of the air mass flux $\dot{m}_a$ (kg/s) and thrust $T$ (kN).
Problem 2: A turbojet with compressor pressure ratio \( P_{rc} = 12 \) is designed to fly at \( M = 1.5 \) at an altitude where the ambient temperature is \( T_a = 220 \) K. The maximum temperature allowed at the entrance of the turbine is \( T_{0t} = 1,800 \) K. In the calculations use \( \gamma = 1.4 \), \( c_p = 1,004 \) J/(kg K), \( R_g = 287 \) J/(kg K), and \( Q_R = 45 \times 10^6 \) J/kg for the properties of the gas and \( \eta_d = 0.97 \), \( \eta_c = 0.85 \), \( \eta_t = 0.92 \), and \( \eta_n = 0.98 \) for the adiabatic efficiencies of the diffuser, compressor, turbine, and nozzle, respectively.

1. Compute the ambient sound speed \( a_a \).
2. Calculate the fuel-to-air ratio \( f \). In the calculation, assume that the burner efficiency is \( \eta_b = 1 \).
3. Find the temperature jump across the turbine \( T_{0s}/T_{0t} \).
4. Determine the pressure jump across the turbine \( p_{0s}/p_{0t} \).
5. Assuming that the pressure drop across the combustor is negligible (i.e. \( r_c = 1 \)), compute the exhaust speed \( u_e \).
6. Obtain the specific thrust \( \tau/\dot{m}_a \), giving the result in (kN \cdot s)/kg.
7. Compute the TSFC, giving the result in kg/(kN \cdot s).
8. Determine the propulsion efficiency \( \eta_p \).
9. Calculate the thermal efficiency \( \eta_{th} \).
10. Obtain the overall efficiency \( \eta_o \).