MAE 113 – Fundamentals of Propulsion – WINTER 2020
MIDTERM EXAM 1 (February 5, 2020)

Problem 1: The thrust of a rocket engine operating at an ambient pressure \( p_a \) can be shown to be

\[
T = \dot{m} u_e + (p_e - p_a) A_e,
\]

where \( \dot{m} \) is the propellant mass flow rate and \( u_e \) and \( p_e \) are the velocity and pressure at the nozzle exit, whose cross section has surface area \( A_e \). Consider a rocket with stagnation pressure \( p_0 \), such that \( p_0/p_a = 20 \) so that the nozzle remains choked. To illustrate the advantages in using a convergent-divergent nozzle, follow the steps indicated below. In the calculations, use \( \gamma = 1.4 \).

For a convergent nozzle with exit section \( A_e = A_t = A_o \):

1. Write the value of \( p_e/p_0 \neq p_a/p_0 \) and \( M_e \) at the nozzle exit. \( p_e/p_0 = p_{CH}/p_0 = 1/1.893, M_e = 1 \)
2. Determine the rocket thrust, expressing the result in the dimensionless form \( T/(p_0 A_e) \). \( T/(p_0 A_e) \approx 1.218 \)
3. The expansion formed at the exit corresponds near the nozzle rim to a Prandtl-Meyer expansion. Obtain the downstream Mach number and flow deflection \( \theta \). \( M_2 = 2.6, \theta = 41.4^\circ \)

Consider now the addition of a divergent section to form a convergent-divergent nozzle with throat area \( A_t = A_o \) and exit area \( A_e > A_o \).

4. Find the value of \( A_e/A_o \) for which \( p_e = p_a \) when \( p_0/p_a = 20 \). \( A_e/A_o \approx 2.9 \)
5. Obtain the corresponding value of the rocket thrust, expressing the result in the dimensionless form \( T/(p_0 A_o) \). \( T/(p_0 A_o) \approx 1.37 \)
Problem 2: The figure below represents the two-dimensional inlet of a supersonic airbreathing engine flying at $M_1 = 3.2$. The air compression is achieved through two oblique shocks induced by a wedge of angle $\alpha = 15^\circ$, as indicated in the figure. To evaluate the drag force $D$ acting on the inlet follow the steps indicated below.

![Supersonic inlet and associated control volume.](image)

1. Calculate the Mach number $M_2$ downstream from the first shock. $M_2 = 2.4$
2. Determine the associated jumps of pressure, density, and sound speed $p_2/p_1$, $\rho_2/\rho_1$, and $a_2/a_1$. $p_2/p_1 = 3, \rho_2/\rho_1 = 2.11, a_2/a_1 = \sqrt{(p_2/p_1)/(\rho_2/\rho_1)} = 1.192$
3. Calculate the Mach number $M_3$ downstream from the second shock. $M_3 = 1.8$.
4. Determine the associated jumps of pressure, density, and sound speed $p_3/p_2$, $\rho_3/\rho_2$, and $a_3/a_2$. $p_3/p_2 = 2.4, \rho_3/\rho_2 = 1.83, a_3/a_2 = 1.145$
5. Use the condition $\dot{m} = \rho_1 u_1 A_1 = \rho_3 u_3 A_3$ to determine the area ratio $A_3/A_1$. $A_3/A_1 = (M_1/M_3)\sqrt{(\rho_1/\rho_3)(p_1/p_3)} = 0.337$
6. The drag force can be evaluated directly by integrating the pressure force acting on the inlet surface to give $D = p_2(A_1 - A_3)$. Use this expression together with the numerical values of $p_2/p_1$ and $A_3/A_1$ obtained above to determine the drag coefficient $C_d = \frac{D}{\frac{1}{2}\rho_1 u_1^2 A_1}$.

$$C_d = \frac{2 \sqrt{\gamma M_1^2 p_2}}{\rho_1 u_1^2 A_1} \left(1 - \frac{A_3}{A_1}\right) = 0.277$$

7. The drag force is related to the variation of pressure and momentum flux of the gas stream entering the engine. Write the horizontal component of the momentum conservation equation in the control volume indicated in the figure, i.e.

$$\int_{S_c} \rho \bar{w} \cdot \bar{n} dS = - \int_{S_c} p m_d dS - D,$$

to show that

$$D = \dot{m}(u_1 - u_3) + A_1 p_1 - A_3 p_3.$$

8. Use the above expression to evaluate $C_d$, verifying that the value obtained is approximately equal to that evaluated earlier in question 6 using the more direct approach. $C_d = 2 \left(1 - \frac{M_1^2}{\gamma M_1^2} \sqrt{\frac{p_2/p_1}{\rho_2/\rho_1}}\right) + \frac{2}{\gamma M_1^2} \left(1 - \frac{A_3}{A_1} \frac{p_3}{p_1}\right) = 0.265$

9. To evaluate the performance of air-breathing engines, inlets are often treated as isentropic (neglecting the presence of oblique shocks). Using the value of $A_3/A_1$ calculated earlier, obtain in that case the value of $M_3, p_3/p_1, \rho_3/\rho_1$, and $a_3/a_1$. $M_3 = 2.03, p_3/p_1 = 6.03, \rho_3/\rho_1 = 3.609, a_3/a_1 = 1.293$

10. Compute the value of $C_d$ for the isentropic inlet. $C_d = 0.216$