A turboprop with peak temperature $T_0 = 1400$ K and compressor pressure ratio $P_{c} = 12$ is to be used for a large cargo airplane with cruise conditions $M = 0.6$ and $T_a = 220$ K. For the calculations below use $\gamma = 1.4$, $\eta_c Q_H/(c_p T_a) = 200$, $a_a = 297$ m/s, and $p_e = p_a$. $\frac{T_{au}}{T_a} = 0.364$

- Obtain the fuel-air ratio $f$ assuming that the adiabatic efficiency of the compressor is $\eta_c = 0.85$.

- Calculate the temperature ratio $T_0e/T_0$ and the pressure ratio $p_0e/p_0$ across the high-pressure (compressor) turbine, whose adiabatic efficiency is $\eta_t = 0.85$.

- Obtain the ideal enthalpy drop available downstream from the turbine $\Delta h = h_{0e} - h_7$, giving the result in the form $\frac{\Delta h}{c_p T_a}$ with

$$\frac{\Delta h}{c_p T_a} = \frac{T_0e}{T_a} \left(1 - \frac{1}{(p_0e/p_a)^{(\gamma-1)/\gamma}} \right).$$

Use $r_c = 1$ and $\eta_d = 0.9$ for the pressure ratio across the combustor and for the adiabatic efficiency of the diffuser, respectively.

- Using $\eta_n = 0.98$ and $\eta_c \eta_d \eta_{pt} = 0.9$ determine the optimal value $\alpha_{opt}$ of the fraction $\alpha$ of available enthalpy to be used in the power turbine.

- For this value $\alpha = \alpha_{opt}$ calculate the specific thrust $\tau/m_a$ and the TSFC of the turboprop.

- Compare the resulting values of $\tau/m_a$ and TSFC with those corresponding to a turbojet with the same efficiencies and the same values of $T_0/T_a$ and $P_{c}$. Comment on the results.

\[ f = \frac{\tau/m_m}{\dot{m}_m} = \frac{\tau}{\dot{m}_m} \left[1 + \frac{P_{c} \frac{P_{c}^{1-\gamma}}{\gamma}}{\eta_{c}} \right] \left[1 + \frac{P_{c} \frac{P_{c}^{1-\gamma}}{\gamma}}{\eta_{c}} \right] = 0.0206 \]

\[ \frac{\Delta h}{c_p T_a} = \frac{T_0e}{T_a} \left(1 - \frac{1}{(p_0e/p_a)^{(\gamma-1)/\gamma}} \right) = \frac{T_0e}{T_a} \left[1 - \frac{1}{(p_0e/p_a)^{(\gamma-1)/\gamma}} \right] = 0.795 \]

\[ \frac{\Delta h}{c_p T_a} = \frac{h_{0e} - h_7}{c_p T_a} = \frac{T_0e}{T_a} \left[1 - \frac{1}{(p_0e/p_a)^{(\gamma-1)/\gamma}} \right] = 0.38 \]

\[ \alpha_{opt} = \frac{1 - \frac{\eta_{c}}{(\eta_{c} \eta_{d} \eta_{pt})^{\frac{1}{2}}} \frac{M^2}{\Delta h/c_p T_a}}{2} = 0.78 \]

\[ \tau/m_m = ma \left[1 + \frac{\tau}{\dot{m}_m} \right] \left(1 + \frac{\tau}{\dot{m}_m} \right) = 0.78 \]

\[ \text{TSFC} = \frac{\tau/m_m}{\dot{m}_m} = 0.0197 \frac{\text{Kgs}}{\text{Kgs}} \]

For a turbojet $\tau = T/m_a$, $\alpha = \frac{T}{m_a} = a \left[1 + \frac{\tau}{\dot{m}_m} \right] \left(1 + \frac{\tau}{\dot{m}_m} \right) = 0.766 \text{ Kgs/Kgs}$
The specific thrust of a Ramjet with \( p_e = p_a \) is given by

\[
\frac{T}{m_a} = \left(1 + f\right)M_e \left(\frac{T_{i1}}{T_a}\right)^{1/2} \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1/2} - M_a \cdot \alpha_e.
\]

in terms of the flight Mach number \( M \), ambient sound speed \( a_a \), peak-to-ambient temperature ratio \( T_{i1}/T_a \), exhaust Mach number \( M_e \), and fuel-air ratio \( f \). The latter can be evaluated from

\[
f = \frac{(T_{i1}/T_a) - \left(1 + \frac{\gamma - 1}{2} M_e^2\right)}{\eta_d Q_{in}/(c_p T_a) - (T_{i1}/T_a)},
\]

as follows from the energy balance across the combustor, whereas the exhaust Mach number \( M_e \) corresponding to a given value of \( M \) depends on the losses of stagnation pressure in the diffuser, combustor, and nozzle.

- Derive an equation for \( M_e \) for known values of the pressure ratio across the combustor \( r_c = p_{a1}/p_{a2} \) and of the adiabatic efficiencies of the diffuser and nozzle \( \eta_d \) and \( \eta_n \).

- Determine the specific thrust of a Ramjet with \( M = 4, T_{i1}/T_a = 10, a_a = 300 \text{ m/s}, \gamma = 1.2, \phi Q_{in}/(c_p T_a) = 220, r_c = 0.95, \eta_d = 0.8, \) and \( \eta_n = 0.95 \). In the calculations use \( \gamma = 1.2 \) for the specific-heat ratio.

- Obtain the corresponding values of TSFC, \( \eta_p \), \( \eta_{th} \), and \( \eta_o \).

\[
\eta_n = \frac{h_u - h_s}{h_u - h_{ss}} = \frac{1 - T_{i1}/T_a}{1 - (P_{a1}/P_{a2})^{\gamma/\gamma} - 1} = 1 - \frac{1}{1 + \frac{\gamma - 1}{2} M_e^2}
\]

so that

\[
\eta_d = \frac{h_{ss} - h_o}{h_{ss} - h_{a}} = \frac{(P_{a2}/P_{a1})^{\gamma - 1}}{1 + \frac{\gamma - 1}{2} M_e^2} = \frac{(P_{a2}/P_{a1})^{\gamma - 1}}{1 + \frac{\gamma - 1}{2} M_e^2} = \frac{(P_{a2}/P_{a1})^{\gamma - 1}}{1 + \frac{\gamma - 1}{2} M_e^2}
\]

\[
\left(1 + \frac{\gamma - 1}{2} M_e^2\right) = \left\{1 - \eta_n \left[1 + \frac{1}{r_c} \left(1 + \eta_d \frac{\gamma - 1}{2} M_e^2\right)\right]\right\}^{-1}
\]

\[
f = 0.0352, \ M_e = 3.356, \ \frac{T}{m_a} = 1.06 \ \frac{\text{km}}{\text{s}}, \ \text{TSFC} = \frac{f}{T/m_a} = 0.332 \ \frac{\text{kg}}{\text{kN} \cdot \text{s}}
\]

\[
\eta_p = 0.728, \ \eta_{th} = 0.501, \ \eta_o = 0.365
\]
Consider the turbojet with afterburner shown in the figure below. Under normal operation the afterburner is off and the specific thrust and TSFC are given by

\[ \frac{T}{m_a} = [(1 + f)(u_e/a_a) - M] a_a \]

and

\[ \text{TSFC} = \frac{m_f}{T} = \frac{f}{T/m_a} \]

where \( f = \dot{m}_f/\dot{m}_a \). The afterburner is used to boost the thrust by burning an additional fuel mass-flow rate \( \dot{m}_f \) downstream from the turbine, resulting in an increased exhaust speed \( u_e^* \). The associated specific thrust and TSFC are given by

\[ \frac{T}{m_a} = [(1 + f^*)(u_e^*/a_a) - M] a_a \]

and

\[ \text{TSFC} = \frac{\dot{m}_f + \dot{m}_f^*}{T} = \frac{f + f^*}{T/\dot{m}_a} \]

where \( f^* = \dot{m}_f^*/\dot{m}_a \).

To investigate the effect of the afterburner, consider an ideal turbojet with \( M = 1.5, \eta_b Q_{R}/(c_p T_a) = 200, a_a = 300 \text{ m/s}, P_{r_e} = 20, T_{0_b}/T_a = 7 \), and \( \gamma = 1.3 \). Assume that the pressure losses in the main combustion chamber and in the afterburner are negligible and that the nozzle area is variable, so that the condition \( p_e = p_a \) is always satisfied.

- Determine the values of \( f \) and \( T_{0_b}/T_a \).

- If the afterburner is off, obtain the values of \( u_e/a_a \) along with the corresponding values of \( T/\dot{m}_a \) and TSFC, \( U_e/a_a = (\frac{3}{2} \frac{T_{0_b}}{T_a}) \left[ (\frac{T_{0_b}}{T_a})^{(\gamma - 1)/\gamma} - \frac{1}{P_{r_e}^{(\gamma - 1)/\gamma}} \right] \frac{1}{P_{r_e}^{1/\gamma}} = 1.30 \frac{\text{km/s}}{\text{m/s}} \).

- When the afterburner is in operation, apply conservation of energy to the associated combustion process to show that

\[ f^* = (1 + f) \left( \frac{T_{0_b}}{T_a} - \left( \frac{T_{0_b}}{T_a} \right)^{\frac{1}{\gamma}} \right) \frac{\dot{m}_f}{\dot{m}_a} \frac{(u_e + u_e^*)}{(u_e + u_e^*)} \frac{h_{05}^*}{h_{05}} \frac{T_{0_b}}{T_a} \]

in terms of the peak downstream temperature \( T_{0_b}/T_a \).

- Obtain \( f^* \) for \( T_{0_b}/T_a = 10 \). \[ f^* = 0.0233 \]

- Analyze the isentropic flow in the nozzle to show that

\[ \frac{u_e^*}{a_a} = \frac{\gamma - 1}{2} \left( \frac{u_e}{a_a} \right)^2 = \frac{T_{0_b}}{T_a} \left( 1 - \frac{T_{0_b}}{T_{0_b}} P_{r_e}^{(\gamma - 1)/\gamma} \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right) \]

- Obtain the value of \( u_e^*/a_a \) along with the corresponding values of \( T/\dot{m}_a \) and TSFC. \[ \frac{u_e^*}{a_a} = 5.987, \frac{T_{0_b}}{T_a} = 1.128 \frac{\text{km/s}}{\text{m/s}}, \text{TSFC} = 0.032 \]