Problem 1 (60 points): The figure below represents schematically the engine of a VTOL aircraft, similar to the Pratt & Whitney F135-PW-600 turbofan engine used in the F-35B. Besides the compressor turbine, the engine has a separate power turbine. In flight mode, this power turbine is used to move the fan, so that the engine operates like a normal turbofan with \( P_r_c = 20, P_r_f = 2, \) and \( B = 8. \) For takeoff and landing, however, the engine switches to a turboshaft mode, with the power turbine used to move an external lift fan. The pressure ratio across the combustor is \( r_c = p_0^4/p_0^3 = 0.9, \) while the adiabatic efficiencies are \( \eta_d = 0.9, \eta_f = 0.85, \eta_c = 0.8, \eta_{ct} = 0.9, \eta_{pt} = 0.9, \eta_n = 0.97, \) and \( \eta_{fn} = 0.97 \) for the diffuser, fan, compressor, compressor turbine, power turbine, nozzle, and fan nozzle, respectively. For the following analysis use \( T_0^4/T_a = 7, (\eta_b Q_R)/(c_p T_a) = 200, a_a = 300 \text{ m/s}, \) and \( \gamma = 1.3. \)

- We begin by analyzing the performance of the engine in flight mode at \( M = 0.8: \)
  1. Obtain the fuel-to-mass ratio \( f. \)
  2. Compute the temperature behind the compressor turbine, giving the result in the form \( T_{0b_a}/T_a. \)
     In the calculation, use the condition \( P_{s_c} = P_{s_{ct}}, \) stating that the compressor turbine is used to move the compressor.
  3. Calculate the pressure behind the compressor turbine, giving the result in the form \( p_{0b_a}/p_a. \)
  4. Determine the temperature and the pressure behind the fan, giving the result in the form \( T_{0f_a}/T_a \) and \( p_{0f_a}/p_a \), respectively.
  5. Assuming that \( p_8 = p_a, \) find the exhaust speed of the bypass stream \( u_{ef}, \) giving the result in the form \( u_{ef}/a_a. \)
  6. Use the condition \( P_{s_f} = P_{s_{pt}} \) to obtain the temperature behind the power turbine, giving the result in the form \( T_{0b_b}/T_a. \)
  7. Calculate the pressure jump across the power turbine, giving the result in the form \( p_{0b_b}/p_a. \)
  8. Assuming that \( p_6 = p_a, \) obtain the exhaust speed of the hot stream \( u_e, \) giving the result in the form \( u_e/a_a. \)
  9. Compute the specific thrust of the turbofan \( T/\dot{m}_a. \)

- Consider now the performance of the engine when operating as a turboshaft during takeoff/landing:
  10. Compute \( f, T_{0b_a}/T_a, \) and \( p_{0b_a}/p_a \) for \( M = 0. \)
  11. Assuming \( p_{0b_b} = p_a, \) determine the shaft power of the power turbine \( P_{s_{pt}} \) (which is used to move the lift fan), giving the result in the nondimensional form \( P_{s_{pt}}/(\dot{m}_a a_a^2). \)
Problem 2 (40 points): The figure below represents schematically a ramjet flying with Mach number \( M = 3 \). In the following analysis, assume that:

(i) The compression through the diffuser can be represented by a normal shock.

(ii) The combustion increases the stagnation temperature by a factor \( T_03/T_02 = 4 \), while keeping the stagnation pressure constant (i.e., \( p_{03} = p_{02} \)).

(iii) The fuel-to-air ratio \( f = \dot{m}_f/\dot{m}_a \) satisfies \( f \ll 1 \).

(iv) The nozzle is ideal and has a throat-to-exit area ratio such that the gas stream is perfectly expanded (i.e. \( p_4 = p_a \)).

1. Calculate the Mach number \( M_2 \) behind the shock.
2. Obtain the stagnation pressure \( p_{02} \) behind the shock, giving the result in the form \( p_{02}/p_a \).
3. Compute the Mach number at the nozzle exit \( M_e = M_4 \).
4. Compute the thrust, given the result in the form \( T/(\dot{m}_a a_a) \), where \( a_a \) is the ambient value of the sound speed.
5. Determine the propulsion efficiency \( \eta_p \).

\[ \begin{array}{c}
\text{1} \quad P_a \\
\downarrow \quad M \\
\text{2} \\
\dot{m}_f \\
\text{3} \\
\text{4} \quad P_a \\
\downarrow \quad M_e 
\end{array} \]