Problem 1: (From Hill and Peterson, pp. 482–483) To illustrate the gains to be made by multistaging, consider a rocket with $u_{eq} = 3,048 \text{ m/s}$, payload $m_{PL} = 1,000 \text{ kg}$, $m_{ST} = 2,000 \text{ kg}$, $m_{PR} = 12,000 \text{ kg}$. Neglecting the effects of gravity and drag on the vehicle acceleration, determine the maximum velocity that can be reached

(a) With a single-stage rocket

(b) With a two-stage rocket. Assume that the structural coefficient $\varepsilon$ and the payload ratio $\lambda$ are identical for both stages.

Problem 2: The space shuttle was a partially reusable human spaceflight vehicle capable of reaching low Earth orbit. The vehicle consisted of a spaceplane for orbit and re-entry ($m_{orbit} = 68 \text{ Tons}$), fueled by a expendable external tank containing the liquid hydrogen and liquid oxygen ($m_{tank} = 26.5 \text{ Tons}$, $m_{LOx-LH2} = 735 \text{ Tons}$), with two reusable strap-on solid rocket boosters (SRB) ($m_{booster} = 91 \text{ Tons}$, $m_{propellant} = 500 \text{ Tons}$). The Space Shuttle had a two-stage ascent. The Shuttle stack launched vertically like a conventional rocket. It lifted off under the power of its two SRB $I_{sp} = 269 \text{ s}$) and three LOx-LH2 rocket engines ($I_{sp} = 455 \text{ s}$). At the end of the first stage $t = t_{b1} = 124 \text{ s}$, the empty boosters were released, and the orbiter and external tank continued to ascend on an increasingly horizontal flight path under power from its main engines. The main engines were shut down at $t = t_{b2} = 480 \text{ s}$, when the maximum velocity $u_2$ was reached. Determine the value of $u_2$ for a payload of $m_{payload} = 25 \text{ Tons}$. Neglect the effect of drag and assume vertical flight during the first stage and an average inclination of 70° from the vertical direction during the second stage. What would be the maximum velocity $u_2$ if the engines were fired sequentially?.

Problem 3: (From Hill and Peterson, Problem 1 p. 559) A rocket is to be designed to produce 5 MN of thrust at sea level. The pressure in the combustion chamber is $p_0 = 7 \text{ MPa}$ and the temperature is $T_0 = 2800 \text{ K}$. If the working fluid is assumed to be a perfect gas with the properties of air at room temperature, determine the following: (a) specific impulse, (b) mass flow rate, (c) throat diameter, (d) exit diameter.

For the same nozzle find (e) thrust at 30-km altitude ($p_a \simeq 1172 \text{ pa}$); (f) thrust at sea level if the chamber pressure were increased to $p_0 = 21 \text{ MPa}$; (g) thrust with properties of water vapor at the same inlet stagnation conditions ($c_p \simeq 3,130 \text{ J/(kg K)}$ and $\gamma = 1.17$); and (h) thrust with stagnation temperature increased to $T_0 = 3600 \text{ K}$. 