For the low-velocity flow in ②–⑤, the Mach number is so small that stagnation properties are almost equal to the static values:

\[ T_0 = T (1 + \frac{\gamma - 1}{2} M^2) = T \] and \[ P_0 = P \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = P. \]

Besides, in the analysis, we assume \( \gamma = \text{constant} \). Air bypass or afterburner are not considered.

The flow decelerates from \( U \) to reach a small velocity.

In the absence of shaft work or heat addition,

\[ h_{O2} = h_{Oa} \rightarrow C_p (T_{O2} - T_a) = \frac{U^2}{2} \rightarrow \frac{T_{O2} - T_a}{T_a} \approx \frac{1}{C_p} \]

Neglecting the presence of upstream shocks (for \( M > 1 \)) and viscous effects in diffuser-

boundary layers \( P_{O2} = P_{Oa} \)

\[
\left(\frac{P_{O2}}{P_{Oa}}\right)^{\gamma \frac{1}{\gamma - 1}} = \left(\frac{P_{Oa}}{P_{O2}}\right)^{\frac{1}{\gamma - 1}} = 1 + \frac{\gamma - 1}{2} M^2
\] (2)

To measure irreversible losses we introduce the adiabatic efficiency, defined as the ratio of the isentropic enthalpy change to reach the compression \( P_{O2}/P_{Oa} \) to the actual enthalpy change:

\[
\eta_a = \frac{h_{O2} - h_{Oa}}{h_{O2} - h_{Oa}} = \frac{T_{O2} - T_a - 1}{T_{O2} - T_a - 1} = \frac{P_{O2}/P_{Oa}}{1 + \frac{\gamma - 1}{2} M^2}
\] (3)

The low-speed stream is dynamically compressed in the compressor.

Because of the presence of moving parts, the flow inside the compressor is unsteady and none of the stagnation properties is conserved. The flow is adiabatic, so that the energy equation yields:

\[
\dot{m}_a (h_{O3} - h_{O2}) = \dot{\mathcal{Q}}_c \rightarrow T_{O3} - T_{O2} = T_3 - T_2 = \frac{\dot{\mathcal{Q}}_c}{C_p \dot{m}_a} \]

If irreversibilities are neglected, the flow can be considered isentropic \( S_3 = S_2 \).

\[
\frac{S_3 - S_2}{C_p} = \ln \left(\frac{T_3/T_2}{P_3/P_2}\right) \rightarrow T_3/T_2 = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma - 1}{\gamma}}
\]

Since \( M_2 = M_1 \) and \( M_3 = M_1 \), we can write

\[
T_{O3}/T_{O2} = \left(\frac{P_{O3}}{P_{O2}}\right)^{\frac{\gamma - 1}{\gamma}}
\] (4)

The change in entropy that occurs in reality is accounted for by introducing the compressor adiabatic efficiency:

\[
\eta_c = \frac{h_{O2} - h_{Oa}}{h_{O2} - h_{Oa}} = \frac{T_{O2} - T_a - 1}{T_{O1} - T_a - 1} = \frac{P_{O2}/P_{Oa}}{P_{O1}/P_{Oa}}
\]

Work per unit mass required to compress the air from \( P_{Oa} \) to \( P_{O2} \):

\[
\eta_c = \frac{T_{O2} (P_{O2}/P_{Oa})}{T_{O1}} = \frac{1}{\eta_c}
\] (5)
After adding fuel, the resulting mixture is burnt with excess air. The
energy balance reads

\[ (\dot{m}_a + \dot{m}_f) h_{h4} - \dot{m}_c h_{h5} = \dot{ma} h_{o3} = \eta Q \dot{m}_f \]

\[
(1 + f) T_{h4} - T_{h3} = \eta_b Q R f \Rightarrow \frac{(1 + f) T_{h4}}{T_a} - \frac{T_{h3}}{T_a} = \eta_b Q R f
\]

\[ f = \frac{T_{h4}/T_a - T_{h3}/T_a}{\eta_b Q R/C_{pTa}} \]

\[ (5) \]

If friction is neglected, then \( P_3 + s_3 u_3^2 = P_4 + s_4 u_4^2 \) \( \Rightarrow \)

\[ \frac{P_{h4}}{P_{h3}} = \frac{P_4}{P_3} \approx 1 \]

(6)

If compressibility \( P_{h4} < P_{h3} \), \( V_c = \frac{P_{h4}}{P_{h3}} < 1 \)

(6.1)

4-5

The flow is expanded dynamically across the turbine to produce the shaft row
needed to move the compressor. \( s_{st} = s_{sc} \). The adiabatic energy balance leads to

\[ (\dot{m}_a + \dot{m}_f) (h_{h4} - h_{o5}) = P_{st} \rightarrow \]

\[ T_{h4} - T_{h5} = T_h - T_5 = \frac{P_{st}}{C_p \dot{m}_a} \]

(7)

Using (7) and (3) with \( s_{st} = s_{sc} \) and \( (\dot{m}_a)_comp = [C_p(\dot{m}_a + \dot{m}_f)]_{turbine} \)

yields

\[ T_{h4} - T_{h5} = T_{h3} - T_{h2} \rightarrow \left[ \frac{T_{h5}}{T_{h4}} = 1 - \frac{T_{h3} - T_{h2}}{T_{h4}} = 1 - \left( \frac{T_{h3} - T_{h2}}{T_{h4}} \right) \frac{T_{h4}}{T_{h4}} \right] \]

(8)

If irreversibilities are neglected, \( s_{st} = s_{sc} \)

\[ \frac{T_5}{T_4} = \left( \frac{P_5}{P_4} \right)^{\gamma - 1} \frac{M_{h4}^{\gamma - 1}}{M_{h4}^{\gamma - 1}} \]

(9)

\[ P_5 = \left( \frac{P_5}{P_4} \right)^{\gamma - 1} \]

\[ T_{h5} = \frac{(P_5)}{C_p T_5} \]

\[ \left[ T_{h5} - T_6 = \frac{U_2^2}{2C_p} \right] \]

(10)

5-6

The flow is expanded adiabatically in the nozzle.

\[ T_{h6} = h_{h6} \]

\[ T_{h6} = T_{h5} \]

(11)

\[ \left[ \frac{P_{h6}}{P_b} \right]^{\gamma - 1} = \left( \frac{P_{h6}}{P_b} \right)^{\gamma - 1} \]

IF IRREVERSIBILITIES ARE NEGLECTED THE FLOW IS ISENTROPIC AND STEADY. ALL STAGNATION PROPERTIES ARE CONSERVED

\[ \frac{T_{h6}}{T_{h5}} = \left( \frac{P_{h6}}{P_b} \right)^{\gamma - 1} = 1 + \frac{\gamma - 1}{2} \frac{M_e^2}{2} \]

\[ \frac{T_{h6}}{T_{h5}} = \frac{P_{h6}}{P_b} \]

\[ \frac{T_{h6}}{T_{h5}} = \frac{P_{h6}}{P_b} \]

\[ \frac{T_{h6}}{T_{h5}} = \frac{P_{h6}}{P_b} \]

\[ \frac{T_{h6}}{T_{h5}} = \frac{P_{h6}}{P_b} \]

\[ \frac{T_{h6}}{T_{h5}} = \frac{P_{h6}}{P_b} \]
The performance of the turbojet can be measured by the specific thrust per unit mass of air mass flow

\[ \frac{f}{m_a} = (1+f) \frac{U_e - U}{m_a} \]

and the thrust specific fuel consumption

\[ TSFC = \frac{m_f}{T/\dot{m}_a} = \frac{f}{T/\dot{m}_a} = \frac{f}{(1+f) \frac{U_e - U}{m_a}} \]

Where \( U = M \sqrt{\gamma R T_a} \), and \( f \) and \( U_e \) can be determined in terms of

\[ \gamma, \sqrt{\gamma R T_a}, \frac{Q_R}{C_p T_a}, \frac{P_{re}}{P_{o2}} \] and \( M \) as follows.

**Ideal Turbojet**

From (5) with \( \eta_b = 1 \)

\[ f = \frac{T_{out}/T_a - T_{in}/T_a}{Q_R/C_p T_a - T_{out}/T_a} = \frac{T_{out}/T_a - \frac{P_{re}}{P_{o2}} \left( 1 + \frac{\gamma - 1}{2} \frac{V_e^2}{T_a} \right)}{Q_R/C_p T_a - T_{out}/T_a} \]

(12)

\[ \frac{U_e^2}{2} = h_{gb} - h_g = C_p T_0 \left( 1 - \frac{T_e}{T_0} \right) \]

(10)

\[ \Rightarrow P_e = \frac{P_{o2}}{P_{o1}} \frac{T_{o2}}{T_{o1}} \]

\[ = C_p T_a \frac{T_05}{T_04} \frac{T_04}{T_a} \left[ 1 - \frac{1}{\left( \frac{T_05}{T_{out}} \right) \frac{P_{re}}{P_{o2}} \left( 1 + \frac{\gamma - 1}{2} \frac{V_e^2}{T_a} \right)} \right] \]

\[ \Rightarrow \frac{T_{05}}{T_{out}} = 1 - \frac{\left( \frac{T_05}{T_{out}} - \frac{T_04}{T_{out}} \right)}{\frac{T_05}{T_{out}}} \]

(8)

\[ \Rightarrow \frac{T_{05}}{T_{out}} = 1 - \left( \frac{1}{\frac{P_{re} - 1}{P_{o2} \left( 1 + \frac{\gamma - 1}{2} \frac{V_e^2}{T_a} \right)}} \right) \]

(13)
From (5)

$$f = \frac{T_{04}/T_a - T_{02}/T_a}{\eta_v Qr / C_p T_a - T_{04}/T_a}$$

and (11r)

$$\left(\frac{u_e^2}{2} = \eta_v C_p T_05 \left[1 - \frac{T_05}{(T_04 / T_05) (\frac{T_05}{T_04})^{\gamma_z \gamma_r}}\right]\right)$$

$$\frac{T_08}{T_04} = \left(\frac{T_02 - 1}{T_02 / T_05 - 1}\right)\frac{T_02}{T_04 / T_05} = 1 - \frac{T_02}{T_04 / T_05}$$

Where

$$\left(\frac{T_08}{T_04} = 1 - \frac{(T_02 - 1) T_02}{T_02 / T_05 - 1}\right) = 1 - \left(1 - \frac{T_{02}}{T_{04}}\right) \frac{P_{02}^{\frac{x_i - 1}{\gamma c}}}{1 + \frac{x_i - 1}{\gamma c}}$$

FIGURE 5.21 Turbojet cruise thrust and fuel consumption (M = 2).

Increasing $T_{04}$ increases significantly $T_{04}/T_a$. The effect on TSPC is less pronounced and depends on $P_{04}$. Maximum thrust and minimum fuel consumption occur for different values of $P_{04}$. As the Mach number increases, the optimum $P_{04}$ decreases.
**TURBOFAN ENGINE**

\[
\frac{T_{\text{in}}}{m_a} = (1 + \epsilon) V_e + B U_{ef} - (1 + B) U, \quad T_{\text{SCF}} = \frac{f}{T_{\text{in}}} (1 + \epsilon) U_e + B U_{ef} - (1 + B) U
\]

**Fan Pressure Ratio**

\[
Pr_f = \frac{P_{07}}{P_{02}}
\]

**Bypass Ratio**

\[
B = \frac{m_{a_{\text{out}}}}{m_{a_{\text{in}}}}
\]

The analysis 3 - 6 is identical to that of the turbojet, except that (8), (14c) & (14r), stemming from \( P_{sc} = P_{sc} \), must be replaced by

\[
P_{sc} = P_{sc} + \rho_s P_t
\]  \( (15) \)

For the bypass flow

**Fan Adiabatic Efficiency**

\[
\eta_f = \frac{h_{07} - h_0}{h_{07} - h_0}
\]

**Fan nozzle adiabatic efficiency**

\[
\eta_{fn} = \frac{h_{07} - h_0}{h_{07} - h_8}
\]

\[ \begin{align*}
0 \rightarrow 7, & \text{ the flow is adiabatic} \\
0 \rightarrow 7, & \text{ from the adiabatic efficiency} \\
7 \rightarrow 8, & \text{ for the fan nozzle} \; h_{07} = h_0
\end{align*} \]

\[
h_{07} - h_8 = h_{08} - h_8 = \frac{U_{ef}^2}{2} = \eta_{fn} h_{07} \left( 1 - \frac{h_8}{h_{07}} \right) = \eta_{fn} C_{pTa} \frac{T_{07} - T_{02}}{T_{a}} \left[ 1 - \frac{1}{\left( \frac{P_{07}}{P_{02}} \right) \left( \frac{P_{02}}{P_{a}} \right)^{\frac{y-1}{2}}} \right]
\]

Using (1), (2r), & (17)

\[
\frac{U_{ef}^2}{2} = \eta_{fn} \frac{yRT_a}{\gamma-1} \left( 1 + \frac{Pr_f^{\frac{y-1}{2}} - 1}{\eta_f} \right) \left( 1 + \frac{y-1}{y} \right) \left[ 1 - \frac{1}{P_{ef}^{\frac{y-1}{2}} \left( 1 + \frac{y-1}{y} \right)} \right]
\]  \( (18) \)
SUBSTITUTING (3), (7) AND (16) INTO (15) WITH THE APPROXIMATION \( \Theta_{r}(\tilde{m}, \tilde{m}_{0}, \Psi) \approx \Theta(\tilde{m}) \)

\[
\begin{align*}
\frac{T_{05} - T_{04}}{T_{05}} &= \frac{T_{05}}{T_{04}} = 1 - \frac{\left(\frac{T_{05}}{T_{02}} - 1\right) + B \left(\frac{T_{07}}{T_{02}} - 1\right)}{T_{04}/T_{a}}
\end{align*}
\]

USING EQUATION (17), AND (17)

\[
\frac{T_{05}}{T_{04}} = 1 - \left[\frac{\left(P_{rc}^{-\frac{1}{2}} - 1\right) + B \left(P_{rc}^{-\frac{1}{2}} - 1\right)}{\gamma_{c}} \right] \left(1 + \frac{\gamma_{c} - 1}{2} M^{2}\right) \left(1 + \frac{\gamma_{c} - 1}{2} M^{2}\right) \left(1 + \frac{\gamma_{c} - 1}{2} M^{2}\right)
\]

THE SOLUTION FOR \( F, U_{e} \) AND \( U_{ef} \) IS OBTAINED FROM (12), (13), (18) AND (19)

IN TERMS OF \( x, T_{04}/T_{a}, \rho, \frac{c_{p}a}{a_{p}T_{a}}, \frac{b_{r}T_{a}}{a_{p}}, P_{rc}, B_{r} \), AND THE DIFFERENT EFFICIENCIES.

Turbofans show clear benefits for subsonic flow. For supersonic flow the large nacelle would cause huge shock-wave losses.

The value of \( \beta \) is limited by structural weight and aerodynamic drag of nacelle and near-tip fan compressibility effects (unless a speed reducer is employed).
TURBOPROP ENGINE

\[ T = T_{jet} + T_{propeller} \]

\[ \frac{T}{m_a} = \frac{(1+f)}{U_e - U} + \frac{\eta_r \eta_g \eta_{pt}}{U m_a} \]  \hspace{1cm} (20)

\[ \Delta h = \text{Available Enthalpy} \]

\[ \alpha = 1 \Rightarrow \text{TURBOSHAFT} \]

POWER TURBINE:

\[ \frac{P_s}{P_{pt}} = (m_{air}m_f)(h_{s5} - h_{o6}) = (m_{air}m_f) \eta_{pt} \Delta h \]  \hspace{1cm} (21)

\[ \eta_{pt} = \frac{h_{o5} - h_{o6}}{h_{o5} - h_{o6}} \]  

\[ \eta_n = \frac{h_{o7} - h_7}{h_{o7} - h_{o6}} \]

\[ h_{o7} - h_7 = \frac{U_e^2}{2} = h_{o6} - h_7 = \eta_n (1 - \alpha) \Delta h \rightarrow U_e = \sqrt{2 \eta_n (1 - \alpha) \Delta h} \]  \hspace{1cm} (22)

SUBSTITUTING (21) & (22) INTO (20) \rightarrow

\[ \frac{T}{m_a} = \frac{(1+f)}{(1-f)} \sqrt{2 \eta_n (1 - \alpha) \Delta h} - U + (1+f) \eta_r \eta_g \eta_{pt} \Delta h \]  \hspace{1cm} (23)

For given flight speed \( U \), available enthalpy \( h \) and efficiencies, the optimum \( \alpha \) for maximum \( T/m_a \) is obtained from

\[ \frac{d(T/m_a)}{d\alpha} = 0 \rightarrow \alpha_{opt} = 1 - \frac{\eta_n}{(\eta_r \eta_g \eta_{pt})^2} \frac{U_e^2}{2 \Delta h} \]  \hspace{1cm} (24a)

GIVING

\[ \left( \frac{T}{m_a} \right)_{\text{max}} = U \left[ (1+f) \eta_r \eta_g \eta_{pt} \Delta h \right] - (1 - \frac{(1+f)}{2} \eta_n \eta_{pt}) \]  \hspace{1cm} (24b)