Problem AB.2.4 The specific thrust of a Ramjet with $p_e = p_a$ is given by

$$\frac{T}{\dot{m}_a} = (1 + f) M_e \left( \frac{T_{0a}}{T_a} \right)^{1/2} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{-1/2} - M $$

in terms of the flight Mach number $M$, ambient sound speed $a_a$, peak-to-ambient temperature ratio $T_{0a}/T_a$, exhaust Mach number $M_e$, and fuel-air ratio $f$. The latter can be evaluated from

$$f = \frac{(T_{0a}/T_a) - (1 + \frac{\gamma - 1}{2} M_e^2)}{\eta_b Q_R/(c_p T_a) - (T_{0a}/T_a)},$$

as follows from the energy balance across the combustor, whereas the exhaust Mach number $M_e$ corresponding to a given value of $M$ depends on the losses of stagnation pressure in the diffuser, combustor, and nozzle.

- Derive an equation for $M_e$ for known values of the pressure ratio across the combustor $r_c = p_{0c}/p_{0a}$ and of the adiabatic efficiencies of the diffuser and nozzle $\eta_d$ and $\eta_n$.

- Determine the specific thrust of a Ramjet with $M = 4$, $T_{0a}/T_a = 10$, $a_a = 300$ m/s, $\eta_b = 1$, $Q_R/(c_p T_a) = 220$, $r_c = 0.95$, $\eta_d = 0.8$, and $\eta_n = 0.95$. In the calculations use $\gamma = 1.2$ for the specific-heat ratio.

- Obtain the corresponding values of TSFC, $\eta_p$, $\eta_{th}$, and $\eta_o$. 

\[ \text{For the nozzle} \quad \eta_n = \frac{\gamma n - 1}{\gamma n - 1} = 1 - \frac{1}{1 + \frac{r_c}{\gamma n}} = \frac{1 - \frac{1}{1 + \frac{r_c}{\gamma n}} M_e^2}{1 - \frac{r_c}{\gamma n}} \]

so that

\[ \text{For the diffuser} \quad \eta_d = \frac{\gamma d - 1}{\gamma d - 1} = 1 - \frac{1}{\gamma d} \left( 1 - \frac{1}{r_c} \right) \left( 1 + \frac{\gamma d}{2} M_e^2 \right)^{-1} \]

\[ \text{In the same calculation} \quad \frac{f}{\gamma n} = 0.0352, \quad M_e = 3.356, \quad \frac{T}{\dot{m}_a} = 1.06 \text{ kN}^2 \text{ kg}^{-2} \text{ s}^{-1}, \quad \text{TSFC} = \frac{f}{\tau L m} = 0.332 \text{ kN}^2 \text{ kg}^{-2} \text{ s}^{-1}, \quad \eta_p = 0.728, \quad \eta_{th} = 0.501, \quad \eta_o = 0.365 \]
Problem AB2.7 The SCRAMJET (supersonic combustion RAMJET) has been considered as a viable propulsion device for future hypersonic planes. It includes an aerodynamic intake that compresses the flow through a series of oblique shocks. The stream, still supersonic, enters then a combustion chamber where chemical heat release takes place. Finally, isentropic acceleration produces a high velocity jet, as needed for propulsion. A simple schematic representation is shown in the figure.

- For a flight Mach number $M = 4$, determine the Mach number $M_2$ and the pressure $p_2/p_a$ at the burner entrance. Assume that the compression is equivalent to that provided by two oblique shocks with deflection $\delta = 18^\circ$.

- Assuming the conditions at the burner exit to be sonic ($M_4 = 1$), determine the pressure $p_4$, giving the result in the form $p_4/p_a$. Use continuity to obtain $T_4/T_a$ (in the calculation, use the approximation $f \ll 1$). Compute the fuel-to-air ratio $f$ for $Q_R/(c_p T_a) = 200$.

- If the burner cross section has a surface area $A_b = 0.23 A$, where $A$ is the exit area, obtain the Mach number $M_e$ and the relative pressure $p_e/p_a$ downstream from the expansion.

- Calculate the thrust, giving the result in the dimensionless form $T/(\dot{m}_a a_a)$.

- The gas expands downstream from the SCRAMJET exit to reach the external ambient pressure $p_a$. Obtain the deflection angle $\theta$ for the resulting Prandtl-Meyer expansion.