Problem AB.1.1 A landing aircraft rolling down the runway at $u = 50 \text{ m/s}$ has an idling turbofan that consumes air at a rate $\dot{m}_a = \dot{m}_{ac} + \dot{m}_{ah} = 100 \text{ kg/s}$ and produces exhaust streams with velocities (relative to the aircraft) $u_{ec} = 100 \text{ m/s}$ and $u_{eh} = 150 \text{ m/s}$. For the analysis, consider that the bypass ratio of the turbofan is $B = \dot{m}_{ac}/\dot{m}_{ah} = 6$ and that the pressure at the exit equals the ambient value.

a) What is the forward thrust of the turbofan during landing? Simplify the expression for $f = \dot{m}_f/\dot{m}_{ah} \ll 1$.

b) What is the magnitude and direction (i.e., forward or reverse) of the thrust if the bypass exhaust is deflected $90^\circ$ without affecting either the air mass flows or the magnitudes of the exhaust velocities?

c) Compute the additional deflection $\alpha$ needed to get zero net thrust once the aircraft has come to a stop. For the new flow conditions, assume that the bypass ratio $B = 6$ and the velocity ratio $u_{ec}^*/u_{eh}^* = 2/3$ remain equal to those found while the aircraft is moving.

Problem AB.1.2 Using the expression

$$T = \dot{m}_{ah} (1 + f) u_{eh} + \dot{m}_{ac} u_{ec} - \dot{m}_a u$$

for the thrust of a turbofan with ambient pressure at the exit, show that the associated thrust specific fuel consumption is

$$\text{TSFC} = \frac{\dot{m}_f}{T} = \frac{f}{(1 + f) u_{eh} + Bu_{ec} - (1 + B)u}$$

where $f = \dot{m}_f/\dot{m}_{ah}$, while the overall efficiency becomes

$$\eta_o = \frac{T u}{\dot{m}_f Q_R} = \frac{(1 + f) u_{eh} + Bu_{ec} - (1 + B)u}{f Q_R u}.$$ 

Introduce the thrust-averaged exhaust velocity

$$\bar{u}_e = \frac{\dot{m}_{ah} (1 + f) u_{eh} + \dot{m}_{ac} u_{ec}}{\dot{m}_{ah} (1 + f) + \dot{m}_{ac}}$$

to derive the approximate result

$$\eta_o \simeq (1 + B) \frac{(\bar{u}_e - u) u}{f Q_R}$$

after using the condition $f \ll 1$. For a given bypass ratio $B$ and a given exhaust velocity $\bar{u}_e$ show that the overall efficiency is maximized for $u = \bar{u}_e/2$. 
Problem AB.1.3 Use Brequet’s range equation

\[ s = \frac{\eta_o Q_R (L/D)}{g} \ln \left( \frac{m_{\text{init}}}{m_{\text{final}}} \right) \]

to show that

\[ \frac{m_{\text{payload}}}{m_o} = \exp \left( - \frac{g s_{\text{max}}}{\eta_o Q_R (L/D)} \right) - \frac{m_{\text{empty}}}{m_o}, \]

where \( m_o \) is the maximum aircraft mass at takeoff. Use the above equation to represent the payload vs range diagram \( (m_{\text{payload}}/m_o - s_{\text{max}} \) with \( s_{\text{max}} \) in km) for an aircraft with \( L/D = 15 \) and \( m_{\text{empty}}/m_o = 0.7 \) whose jet engine has an overall efficiency \( \eta_o = 0.6 \). Use the value \( Q_R = 45,000 \) kJ/kg, typical of hydrocarbon fuels, in the evaluation.

Problem AB.1.4 A turbojet is flying at \( M = 1.3 \) at an altitude where \( T_a = 200 \) K. At the exhaust \( p_e = p_a, \ M_e = 1.6, \) and \( T_e = 1100 \) K. The fuel-to-air ratio is \( f = 0.06 \) and the heat of reaction is \( Q_R = 45,000 \) kJ/kg. Assuming that the gas behaves as air at normal ambient conditions (i.e., \( \gamma = 1.4 \) and \( R_g = 287 \) J/(kg K)), compute:

- The specific thrust \( T/\dot{m}_a \). \( T/\dot{m}_a \simeq 0.759 \) (kN \cdot s)/kg
- The TSFC. TSFC\( \simeq 0.079 \) kg/(kN \cdot s)
- The propulsion efficiency. \( \eta_p \simeq 0.526 \)
- The thermal efficiency. \( \eta_{th} \simeq 0.1979 \)
- The overall efficiency. \( \eta_o \simeq 0.1036 \)
Problem AB.2.1 Consider the performance of a Ramjet. Using

\[
\frac{T}{\dot{m}_a} = M \sqrt{\gamma RT_a} \left[ (1 + f) \sqrt{\frac{T_{0a}}{T_a}} \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{-1/2} - 1 \right]
\]

together with

\[
f = \frac{T_{0a}}{T_a} - \frac{T_{02}/T_a}{Q_R/(c_p T_a) - \dot{m}_a/T_a}
\]

and the definition \(\eta_o = T u/(Q_R \dot{m}_f)\), derive explicit expressions for the thrust per unit mass flow of air \(T/\dot{m}_a\), thrust specific fuel consumption TSFC = \(\dot{m}_f/T\), and overall efficiency \(\eta_o\) of an ideal ramjet in terms of \(a_a = \sqrt{\gamma RT_a}\), \(Q_R/(\gamma c_p T_a)\), \(T_{0a}/T_a\), and the flight Mach number \(M\), verifying that the result can be written in the form

\[
\frac{T}{\dot{m}_a} = M \sqrt{\gamma RT_a} \left[ \sqrt{\frac{T_{0a}}{T_a}} - \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} \right] \frac{Q_R/(\gamma c_p T_a)}{\left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} - 1}
\]

TSFC = \[
\frac{Q_R/(\gamma c_p T_a)}{M \sqrt{\gamma RT_a} \left[ \sqrt{\frac{T_{0a}}{T_a}} + \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} \right]}
\]

and

\[
\eta_o = \frac{(\gamma - 1) M^2 \left[ \sqrt{\frac{T_{0a}}{T_a}} + \left( \frac{Q_R}{\gamma c_p T_a} \right) / \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} \right]}{[\gamma c_p T_a] \left[ \sqrt{\frac{T_{0a}}{T_a}} + \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} \right]}
\]

Use these results to determine an explicit expression for the value of \(M\) at which the thrust vanishes as well as the corresponding values of TSFC and \(\eta_o\). For the case \(a_a = 300 \text{ m/s}, Q_R/(\gamma c_p T_a) = 200\), and \(T_{0a}/T_a = 12\), obtain numerically the value of \(M\) at which the \(T/\dot{m}_a\) reaches its peak value. Obtain also this peak value, giving the result in (kN s)/kg, as well as the accompanying values of \(\eta_o\) and TSFC, with the latter expressed in kg/(kN s). \(T/\dot{m}_a \approx 1.07 (\text{kN s})/\text{kg}, \eta_o \approx 0.3564, \text{TSFC} \approx 0.048 \text{ kg/(kN s)}\)

Problem AB.2.2 Ramjet diffusers typically employ pointy center bodies to slow down the incoming flow through a series of oblique shocks. As an alternative, investigate the use of a blunt center body, as shown in the figure. Assuming that the bow shock that emerges can be approximated as a normal shock wave, obtain the jump in stagnation pressure \(r_d = p_{02}/p_{0a}\) as a function of the flight Mach number \(M\). Compare the solution with the expression \(r_d = 1 - 0.1(M - 1)^{3/2}\) that describes the performance of typical ramjet diffusers (according to the Aircraft Industries Association). Does the blunt-body alternative appear to be an advantageous configuration?
Problem AB.2.3 Plot the variation with flight Mach number of the propulsion and thermal efficiencies of a Ramjet operating at high altitude \((T_a = 200\, \text{K})\). The nozzle inlet temperature is \(T_{04} = 3000\, \text{K}\). In the evaluation assume that the Ramjet is ideal and that \(Q_R = 45 \times 10^6\, \text{J/kg}\), \(\gamma = 1.3\), and \(c_p = 1000\, \text{J/(kg K)}\).

Problem AB.2.4 The specific thrust of a Ramjet with \(p_e = p_a\) is given by

\[
\frac{T}{m_a} = \left(1 + f\right) M_e \left(\frac{T_{04}}{T_a}\right)^{1/2} \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1/2} - M \right] a_a,
\]

in terms of the flight Mach number \(M\), ambient sound speed \(a_a\), peak-to-ambient temperature ratio \(T_{04}/T_a\), exhaust Mach number \(M_e\), and fuel-air ratio \(f\). The latter can be evaluated from

\[
f = \frac{(T_{04}/T_a) - (1 + \frac{\gamma - 1}{2} M^2)}{\eta_b(Q_R/(c_p T_a))} - (T_{04}/T_a),
\]

as follows from the energy balance across the combustor, whereas the exhaust Mach number \(M_e\) corresponding to a given value of \(M\) depends on the losses of stagnation pressure in the diffuser, combustor, and nozzle.

- Derive an equation for \(M_e\) for known values of the pressure ratio across the combustor \(r_c = p_{04}/p_{02}\) and of the adiabatic efficiencies of the diffuser and nozzle \(\eta_d\) and \(\eta_n\).
- Determine the specific thrust of a Ramjet with \(M = 4\), \(T_{04}/T_a = 10\), \(a_a = 300\, \text{m/s}\), \(\eta_b = 1\), \(Q_R/(c_p T_a) = 220\), \(r_c = 0.95\), \(\eta_d = 0.8\), and \(\eta_n = 0.95\). In the calculations use \(\gamma = 1.2\) for the specific-heat ratio. \(T/m_a \simeq 1.06\, (\text{kN} \cdot \text{s})/\text{kg}\)
- Obtain the corresponding values of TSFC, \(\eta_p\), \(\eta_{th}\), and \(\eta_o\). \(\text{TSFC} \simeq 0.0332\, \text{kg/(kN} \cdot \text{s)}\), \(\eta_p \simeq 0.728\), \(\eta_{th} \simeq 0.501\), and \(\eta_o \simeq 0.365\).
Problem AB.2.5 The figure below represents schematically a ramjet flying at Mach number $M = 3$. In the following analysis, assume that:

(i) The compression through the diffuser can be represented by a normal shock.
(ii) The combustion increases the stagnation temperature by a factor $T_0/\bar{T}_0 = 4$, while keeping the stagnation pressure constant (i.e., $p_0 = p_{02}$).
(iii) the fuel-to-air ratio $f = \dot{m}_f/\dot{m}_a$ satisfies $f \ll 1$.
(iv) the nozzle is ideal and has a throat-to-exit area ratio such that the gas stream is perfectly expanded (i.e. $p_4 = p_a$).

1. Calculate the Mach number $M_2$ behind the shock. $M_2 \simeq 0.475$
2. Obtain the stagnation pressure $p_{02}$ behind the shock, giving the result in the form $p_{02}/p_a$. 
   $p_{02}/p_a \simeq 12.056$
3. Compute the Mach number at the nozzle exit $M_e = M_4$. $M_e \simeq 2.2766$
4. Compute the thrust, given the result in the form $T/(\dot{m}_a a_a)$, where $a_a$ is the ambient value of the sound speed. $T/(\dot{m}_a a_a) \simeq 2.3387$
5. Determine the propulsion efficiency $\eta_p$. $\eta_p \simeq 0.7195$

Problem AB.2.6 In supersonic flight, most of the irreversibility associated with the flow through the diffuser comes from the presence of shock waves, while the effect of viscous dissipation in the diffuser-wall boundary layers is of lesser importance. Assuming that a single normal shock is standing in front of the inlet and that the effect of the boundary layers is entirely negligible, obtain an explicit analytic expression for the adiabatic efficiency of the diffuser $\eta_d$ in terms of the flight Mach number $M > 1$ and the specific heat ratio $\gamma$. 

![Diagram of a ramjet with labels and arrows indicating the flow paths.](image)
Problem AB.2.7 The SCRAMJET (supersonic combustion RAMJET) has been considered as a viable propulsion device for future hypersonic planes. It includes an aerodynamic intake that compresses the flow through a series of oblique shocks. The stream, still supersonic, enters then a combustion chamber where chemical heat release takes places. Finally, isentropic acceleration produces a high velocity jet, as needed for propulsion. A simple schematic representation is shown in the figure.

- For a flight Mach number $M = 4$, determine the Mach number $M_2$ and the pressure $p_2/p_a$ at the burner entrance. Assume that the compression is equivalent to that provided by two oblique shocks with deflection $\delta = 18^\circ$. $M_2 \simeq 1.99$, $p_2/p_a \simeq 13.9$

- Assuming the conditions at the burner exit to be sonic ($M_4 = 1$), determine the pressure $p_4$, giving the result in the form $p_4/p_a$. Use continuity to obtain $T_4/T_a$ (in the calculation, use the approximation $f \ll 1$). Compute the fuel-to-air ratio $f$ for $Q_R/(c_p T_a) = 200$. $p_4/p_a \simeq 37.95$, $T_4/T_a \simeq 4.40$

- If the burner cross section has a surface area $A_b = 0.23 A$, where $A$ is the exit area, obtain the Mach number $M_e$ and the relative pressure $p_e/p_a$ downstream from the expansion. $M_e \simeq 3.02$, $p_e/p_a \simeq 1.90$

- Calculate the thrust, giving the result in the dimensionless form $T/(\dot{m}_a a_a)$.

- The gas expands downstream from the SCRAMJET exit to reach the external ambient pressure $p_a$. Obtain the deflection angle $\theta$ for the resulting Prandtl-Meyer expansion. $\theta \simeq 7.1^\circ$
Problem AB.2.8 One concept for hypersonic air-breathing propulsion is a de tonative-combustion RAMJET, in which fuel added to the air burns in a detonation wave. As a simplified model for analysis of such a device, assume that the standing detonation can be modeled as a normal shock wave followed by a region of constant-area heat addition that leaves the flow in sonic conditions \((M_4 = 1)\) and that the downstream divergent nozzle expands the stream to ambient pressure \((p_e = p_a)\). In your calculations use \(\gamma = 1.4\) for the specific heat ratio.

1. Determine the Mach number behind the shock wave \(M_2\) for \(M = 5\). \(M_2 \approx 0.415\)
2. Obtain the associated pressure and temperature ratios \(p_2/p_a\) and \(T_2/T_a\). \(p_2/p_a \approx 29, T_2/T_a \approx 5.8\)
3. Compute the pressure after combustion is completed, giving the result in the form \(p_4/p_a\). \(p_4/p_a \approx 15\)
4. Calculate the associated post-combustion temperature, giving the result in the form \(T_4/T_a\). \(T_4/T_a \approx 9.0\)
5. Compute the Mach number at the nozzle exit \(M_e\). \(M_e \approx 2.83\)
6. Find the exit-to-entrance temperature ratio \(T_e/T_a\). \(T_e/T_a \approx 4.15\)
7. Determine the exhaust speed \(u_e\), giving the result in the form \(u_e/a_a\). \(u_e/a_a \approx 5.76\)
8. Calculate the needed exit-to-combustor area ratio \(A_e/A_c\). \(A_e/A_c \approx 3.6\)
9. Obtain the specific thrust for this engine \(T/\dot{m} = u_e - u\), giving the result in the form \(T/(\dot{m}a_a)\), where \(a_a\) is the ambient sound speed. \(T/(\dot{m}a_a) \approx 0.76\)
10. Find the propulsion efficiency \(\eta_p \approx 0.929\)

\[
\eta_p = \frac{T u}{(m/2)(u_e^2 - u^2)}
\]

Problem AB.2.9 An ideal ramjet is flying at \(M = 3\) at an altitude where \(T_a = 200\) K. The maximum temperature is \(T_0 = 2,600\) K and the heat of reaction is \(Q_R = 45,000\) kJ/kg. Assuming that the gas behaves as air at normal ambient conditions (i.e., \(\gamma = 1.4\) and \(R_g \approx 287\) J/(kg K) with \(c_p = R_g \gamma / (\gamma - 1)\)) and that the burning efficiency is \(\eta_b = 1\), compute:

1. The fuel-to-air ratio \(f\). \(f = 0.0483\)
2. The exit speed \(u_e\). \(u_e = 1,832.5\) m/s
3. The specific thrust \(T/\dot{m}\) in \(\text{kN s/kg}\). \(T/\dot{m} = 1.07\) kN s/kg
4. The TSFC in \(\text{kg/(kN s)}\) \(\text{TSFC} = 0.0452\) kN s/kg
5. The overall efficiency \(\eta_o\). \(\eta_o = 0.4185\)
Problem AB.3.1 Determine the compressor pressure ratio that maximizes the specific thrust of an ideal turbojet for given values of the turbine inlet-to-ambient temperature ratio $T_0/\bar{T}_a$ and the flight Mach number $M$. Use the approximation $f \ll 1$ in the derivation. Obtain also an expression for the value of $M$ above which a compressor is no longer needed to provide optimum specific thrust. Calculate this limiting value of $M$ for the case $T_0 = 1700\text{ K}$, $\bar{T}_a = 220\text{ K}$, and $\gamma = 1.4$. $M \approx 3.09$

Problem AB.3.2 The Concorde Rolls-Royce Olympus 593 engines were designed to cruise at $M = 2$ at an altitude of 53,000 feet ($\bar{T}_a = 216\text{ K}$). At the time, the maximum turbine inlet temperature was limited to $T_0 = 1350\text{ K}$. The engine employed a compressor with pressure ratio $p_{rc} = 11.3$. In trying to understand the selection of the latter value it is of interest to determine the variation of $T/\dot{m}_a$, TSFC, and $\eta_o$ with $p_{rc}$ for the Olympus 593 engine, assuming the following values for the efficiencies and combustor pressure ratio: $\eta_m = 0.97$, $\eta_d = 0.94$, $\eta_c = 0.80$, $\eta_t = 0.88$, $\eta_b = 1.0$, and $r_c = 1.0$. In the calculation, use $\gamma = 1.4$, $R = 287\text{ J/(kg K)}$, and $Q_R = 45 \times 10^6\text{ J/kg}$.

Problem AB.3.3 Two options are being considered for the propulsion device of a long-range missile designed to fly at a cruise Mach number $M = 2.0$ at an altitude where $\bar{T}_a = 220\text{ K}$:

- A Ramjet with peak temperature $T_0 = 2700\text{ K}$
- A Turbojet with compressor pressure ratio $p_{rc} = 30$ and peak temperature $T_0 = 1800\text{ K}$

The nozzle in both designs will have the exit area required to match the ambient pressure at the exit section $(p_e = p_a)$. Because weight is a key issue, the preferred option is the one that provides a higher specific thrust $T/\dot{m}_a$. In the calculation, assume that the two engines behave as ideal and that the gas is a perfect gas with $\gamma = 1.4$ and $R = 287\text{ J/(kg K)}$. In both cases, use $Q_R = 45 \times 10^6 \text{ J/kg}$ for the fuel heat of reaction.

1. Determine the values $T/\dot{m}_a$ for both engines. Based on the result, select the propulsion device for the application at hand.

2. Obtain in both cases the associated thrust specific fuel consumption TSFC.

Problem AB.3.4 Two options are being considered for the propulsion device of a long-range missile designed to fly at a cruise Mach number $M = 2.0$ at an altitude where $\bar{T}_a = 220\text{ K}$:

- A Ramjet with peak temperature $T_0 = 2900\text{ K}$
- A Turbojet with compressor pressure ratio $p_{rc} = 20$ and peak temperature $T_0 = 1800\text{ K}$

The nozzle in both designs will have the exit area required to match the ambient pressure at the exit section $(p_e = p_a)$. Because weight is a key issue, the preferred option is the one that provides a higher specific thrust $T/\dot{m}_a$. In the calculation, assume that the two engines behave as ideal and that the gas is a perfect gas with $\gamma = 1.4$ and $R = 287\text{ J/(kg K)}$. In both cases, use $Q_R = 45 \times 10^6 \text{ J/kg}$ for the fuel heat of reaction.

1. Determine the values $T/\dot{m}_a$ for both engines. Based on the result, select the propulsion device for the application at hand. $T/\dot{m}_a \approx (1.11, 0.73)\text{ (kN \cdot s)/kg}$

2. Obtain in both cases the associated thrust specific fuel consumption TSFC. $\text{TSFC} \approx (0.054, 0.027)\text{ kg/(kN \cdot s)}$
Problem AB.3.5 Two options are being considered for the propulsion device of an experimental high-speed aircraft designed to fly at a cruise Mach number $M = 2.0$ at an altitude where $T_a = 200$ K

- A Ramjet.
- A Turbojet with compressor pressure ratio $P_{rc} = 20$.

The nozzle in both designs will have the exit area required to match the ambient pressure at the exit section ($p_e = p_a$). Because weight is a key issue, the option preferred is the one that provides a higher specific thrust $T/\dot{m}_a$. In the calculation of the thrust use the approximation $f \ll 1$ and assume that the two engines behave as ideal and that the gas properties are $\gamma = 1.4$ and $R = 287 \text{ J/(kg K)}$.

1. Determine the values of $T/\dot{m}_a$ for both engines using $T_{04} = 1800$ K for the peak temperature. Based on the result, select the propulsion device for the application at hand. $T/\dot{m}_a \simeq (0.7, 0.768) \text{ (kN \cdot s)/kg}$

2. The maximum temperature in ramjets can be significantly higher (why?). Redo the computation of the specific thrust of the ramjet with $T_{04} = 2500$ K and comment on the result. $T/\dot{m}_a \simeq 0.927 \text{ (kN \cdot s)/kg}$

Problem AB.3.6 The Concorde range was only $s_{\text{CON}} = 4,000$ miles, barely enough to cross the Atlantic (3,600 miles) but not enough for trans-Pacific operation, which is of primary interest nowadays. You are the chief engineer for the design of a new supersonic passenger aircraft, based on the Concorde, which must be able to operate the San Francisco-Tokyo route (5,200 miles). The first question you need to answer is whether the technical improvements in turbomachinery efficiency and turbine-blade materials that have occurred over the last forty years are by themselves sufficient to increase the range by the necessary amount, maintaining the flight conditions of the original plane ($M = 2$ and $T_a = 216$ K). Note that, if one assumes that the aerodynamic characteristics and the mass distribution of the new aircraft are identical to those of the Concorde (i.e., same values of $L/D$ and $m_{\text{fuel}}/(m_{\text{payload}} + m_{\text{struc}})$) then the new range will be given simply by $s_{\text{NEW}} = (\eta_{\text{NEW}}/\eta_{\text{CON}}) s_{\text{CON}}$

in terms of the overall engine efficiencies of the original Concorde Rolls-Royce Olympus 593 engine, $\eta_{\text{CON}}$, and of the new turbojet, $\eta_{\text{NEW}}$. To compute these two overall efficiencies, use $\gamma = 1.4$, $R = 287 \text{ J/(kg K)}$, and $Q_R = 45 \times 10^6 \text{ J/kg}$ and assume that the burning efficiency is unity ($\eta_b = 1$) and that no pressure loss occurs in the combustor $(r_c = p_{04}/p_{03} = 1)$. For the Olympus 593, the peak turbine-inlet temperature was $T_{04} = 1350$ K, whereas the combination of new metal alloys and advanced blade cooling techniques enables the temperature of the new design to be increased to $T_{04} = 1750$ K. Besides, the adiabatic efficiencies of the different engine elements have increased over time from the values $\eta_d = 0.94$, $\eta_c = 0.80$, $\eta_r = 0.88$ and $\eta_n = 0.97$ of the Olympus 593 engine to the new high-performance values $\eta_d = 0.97$, $\eta_c = 0.88$, $\eta_r = 0.93$ and $\eta_n = 0.99$. To minimize fuel consumption, in the new design the compressor pressure ratio, which was $p_{rc} = 11.3$ for the Concorde, should be increased to $p_{rc} = 40$. Calculate the range of the new supersonic aircraft. Would it be able to fly nonstop the San Francisco-Tokyo route? $s_{\text{NEW}} \simeq 4,724$ miles
Problem AB.3.7 Consider the turbojet with afterburner shown in the figure below. Under normal operation the afterburner is off and the specific thrust and TSFC are given by

\[
\frac{T}{\dot{m}_a} = [(1 + f)\left(\frac{u_e}{a_a}\right) - M] a_a
\]

and

\[
\text{TSFC} = \frac{\dot{m}_f}{T} = \frac{f}{T/\dot{m}_a},
\]

where \( f = \dot{m}_f/\dot{m}_a \). The afterburner is used to boost the thrust by burning an additional fuel mass-flow rate \( \dot{m}^*_f \) downstream from the turbine, resulting in an increased exhaust speed \( u^*_e \). The associated specific thrust and TSFC are given by

\[
\frac{T}{\dot{m}_a} = [(1 + f + f^*)\left(\frac{u^*_e}{a_a}\right) - M] a_a
\]

and

\[
\text{TSFC} = \frac{\dot{m}_f + \dot{m}^*_f}{T} = \frac{f + f^*}{T/\dot{m}_a},
\]

where \( f^* = \dot{m}^*_f/\dot{m}_a \).

To investigate the effect of the afterburner, consider an ideal turbojet with \( M = 1.5, \eta_b Q_R/(c_p T_a) = 200, a_a = 300 \text{ m/s}, P_{r_e} = 20, T_{04}/T_a = 7, \) and \( \gamma = 1.3 \). Assume that the pressure losses in the main combustion chamber and in the afterburner are negligible and that the nozzle area is variable, so that the condition \( p_e = p_a \) is always satisfied.

• Determine the values of \( f \) and \( T_{05}/T_{04} \). \( f \approx 0.0224, T_{05}/T_{04} \approx 0.81 \)

• If the afterburner is off, obtain the values of \( u_e/a_a \) along with the corresponding values of \( T/\dot{m}_a \) and TSFC. \( u_e/a_a \approx 4.5, T/\dot{m}_a \approx 0.930 \text{ (kN \cdot s)/kg}, \text{ TSFC} \approx 0.024 \text{ kg/(kN \cdot s)} \)

• When the afterburner is in operation, apply conservation of energy to the associated combustion process to show that

\[
f^* = (1 + f) \left( \frac{T_{05}}{T_a} - \left( \frac{T_{05}}{T_{04}} \right) \left( \frac{T_{04}}{T_a} \right) \right)
\]

in terms of the peak downstream temperature \( T^*_{05}/T_a \).

• Obtain \( f^* \) for \( T_{05}/T_a = 10 \). \( f^* \approx 0.0233 \)

• Analyze the isentropic flow in the nozzle to show that

\[
\frac{\gamma - 1}{2} \left( \frac{u^*_e}{a_a} \right)^2 = \frac{T^*_{05}}{T_a} \left( 1 - \frac{T_{05}}{T_{04}} P_{r_e}^{(\gamma-1)/\gamma} \right) \left( 1 + \frac{\gamma - 1}{2} M^2 \right).
\]

• Obtain the value of \( u^*_e/a_a \) along with the corresponding values of \( T/\dot{m}_a \) and TSFC. \( u^*_e/a_a \approx 5.987, T/\dot{m}_a \approx 1.428 \text{ (kN \cdot s)/kg}, \text{ TSFC} \approx 0.032 \text{ kg/(kN \cdot s)} \)
**Problem AB.3.8** A turbojet engine has a compressor pressure ratio $P_{rc} = 30$ and a maximum temperature at the end of the combustor $T_{04} = 1700$ K. The adiabatic efficiencies of the diffuser, compressor, turbine, and nozzle are $\eta_d = 0.9$, $\eta_c = 0.9$, $\eta_t = 0.8$, and $\eta_n = 0.9$, respectively. The pressure loss in the combustor is negligible and the combustion efficiency if $\eta_b = 1$. For a flight Mach number $M = 0.8$ and ambient temperature $T_a = 205$ K, determine the specific thrust $T/\dot{m}$, TSFC, and overall efficiency of the engine.

**Problem AB.3.9** Show that for an ideal turbojet ($\eta_d = \eta_c = r_c = \eta_b = \eta_t = \eta_n = 1$) at takeoff ($M = 0$) the fuel-to-air ratio $f$ and the exhaust jet speed $u_e$ are given by

$$f = \frac{T_{04}/T_a - P_{rc}^{(\gamma-1)/\gamma}}{Q_R/(c_pT_a) - T_{04}/T_a}$$

and

$$\left(\frac{\gamma - 1}{2}\right)\left(\frac{u_e}{a_a}\right)^2 = \frac{(P_{rc}^{(\gamma-1)/\gamma} - 1)(T_{04}/T_a - P_{rc}^{(\gamma-1)/\gamma})}{P_{rc}^{(\gamma-1)/\gamma}},$$

respectively. Use this last equation to show that, in the limit $f \ll 1$, the specific thrust at takeoff can be expressed in the form

$$\frac{T}{\dot{m}_a} = \left(\frac{\gamma - 1}{2}\right)^{-1/2} \left[\frac{(P_{rc}^{(\gamma-1)/\gamma} - 1)(T_{04}/T_a - P_{rc}^{(\gamma-1)/\gamma})}{P_{rc}^{(\gamma-1)/\gamma}}\right]^{1/2} a_a.$$

Show that the specific thrust takes a maximum value

$$\frac{T}{\dot{m}_a} = \left(\frac{\gamma - 1}{2}\right)^{-1/2} \left[\left(\frac{T_{04}}{T_a}\right)^{2(\gamma-1)} - 1\right] a_a$$

for a compressor pressure ratio $P_{rc} = (T_{04}/T_a)^{2(\gamma-1)}$. 
Problem AB.3.10 A turbojet with compressor pressure ratio $P_{rc} = 12$ is designed to fly at $M = 1.5$ at an altitude where the ambient temperature is $T_a = 220$ K. The maximum temperature allowed at the entrance of the turbine is $T_{04} = 1,800$ K. In the calculations use $\gamma = 1.4$, $c_p = 1,004$ J/(kg K), $R_g = 287$ J/(kg K), and $Q_R = 45 \times 10^6$ J/kg for the properties of the gas and $\eta_d = 0.97$, $\eta_c = 0.85$, $\eta_t = 0.92$, and $\eta_n = 0.98$ for the adiabatic efficiencies of the diffuser, compressor, turbine, and nozzle, respectively.

1. Compute the ambient sound speed $a_a$. $a_a \approx 297.3$ m/s

2. Calculate the fuel-to-air ratio $f$. In the calculation, assume that the burner efficiency is $\eta_b = 1$. $f \approx 0.0254$

3. Find the temperature jump across the turbine $T_{05}/T_{04}$. $T_{05}/T_{04} \approx 0.784$

4. Determine the pressure jump across the turbine $p_{05}/p_{04}$. $p_{05}/p_{04} \approx 0.392$

5. Assuming that the pressure drop across the combustor is negligible (i.e. $r_c = 1$), compute the exhaust speed $u_e$. $u_e \approx 1,239$ m/s

6. Obtain the specific thrust $T/\dot{m}_a$, giving the result in (kN · s)/kg. $T/\dot{m}_a \approx 0.824$ (kN · s)/kg

7. Compute the TSFC, giving the result in kg/(kN · s). TSFC$ \approx 0.0308$ kg/(kN · s)

8. Determine the propulsion efficiency $\eta_p$. $\eta_p \approx 0.534$

9. Calculate the thermal efficiency $\eta_{th}$. $\eta_{th} \approx 0.602$

10. Obtain the overall efficiency $\eta_o$. $\eta_o \approx 0.321$
Problem AB.3.11 Consider a turbojet with compressor pressure ratio $P_{cr} = 20$, nozzle exit area $A_e = 0.5$ m², peak temperature $T_{0a} = 1800$ K, and adiabatic efficiencies (diffuser, compressor, turbine, and nozzle) $\eta_d = 0.85$, $\eta_c = 0.8$, $\eta_t = 0.9$, and $\eta_n = 1$ (i.e., the nozzle is assumed to be ideal). Follow the steps listed below to calculate the air mass flux

$$\dot{m}_a = \frac{\rho_e u_e A_e}{1 + f} = \frac{\gamma}{1 + f} p_a A_e M_e \left( \frac{p_e}{p_a} \right) \left( \frac{T_e}{T_a} \right)^{-1/2}$$

and the thrust

$$T = \dot{m}_a a_a [(1 + f) u_e/a_a - M] + A_e (p_e - p_a)$$

both at cruise conditions (36,000 feet) and at takeoff. In the calculations, assume that the effective heat of reaction is $\eta_b Q_R = 45 \times 10^6$ J/kg, that the pressure loss in the combustor is negligible (i.e., $r_c = p_{0c}/p_{0a} = 1$), and that $\gamma = 1.4$ and $R_g = 287$ J/(kg K), corresponding to $c_p = 1,004.5$ J/(kg K).

During flight at cruise conditions ($M = 1.5$, $p_a = 22,600$ Pa, and $T_a = 217$ K):

1. Determine the fuel-to-air mass ratio $f$. $f \simeq 0.0222$
2. Obtain the temperature jump across the turbine $T_{0e}/T_{0a}$. $T_{0e}/T_{0a} \simeq 0.704$
3. Assuming that the nozzle is dimensioned to give $p_e = p_a$ at cruise, calculate the exit Mach number $M_e$, along with the exit temperature $T_e/T_a$ and the exit velocity $u_e/a_a$. $M_e \simeq 2.43$, $T_e/T_a \simeq 2.68$, $u_e/a_a \simeq 3.98$
4. Compute the nozzle throat-to-exit area ratio $A_t/A_e$. $A_t/A_e \simeq 0.405$
5. Determine the air mass flux $\dot{m}_a$ (kg/s) and the thrust $T$ (kN). $\dot{m}_a \simeq 77.8$ kg/s, $T \simeq 59$ kN

At takeoff ($M = 0$, $p_a = 101,300$ Pa, and $T_a = 288$ K):

6. Determine the fuel-to-air mass ratio $f$. $f \simeq 0.0238$
7. Obtain the temperature jump across the turbine $T_{0e}/T_{0a}$ and the pressure behind the turbine, giving the latter in the form $p_{0e}/p_a$. $T_{0e}/T_{0a} \simeq 0.729$, $p_{0e}/p_a \simeq 5.70$
8. The nozzle has fixed geometry (i.e., the value of $A_t/A_e$ is that computed above). Assuming that the nozzle remains choked at takeoff, with an oblique shock forming right outside the nozzle exit, obtain the values of the Mach number $M_e$, temperature $T_e/T_a$, velocity $u_e/a_a$, and pressure $p_e/p_a$ at the nozzle exit section. $M_e \simeq 2.42$, $T_e/T_a \simeq 2.09$, $u_e/a_a \simeq 3.51$, $p_e/p_a \simeq 0.372$
9. Obtain the deflection of the jet stream across the oblique shock near the nozzle rim $\delta$, as well as the corresponding post-shock Mach number $M_d$. $\delta \simeq 17.5^\circ$, $M_d \simeq 1.75$
10. Compute the takeoff values of the air mass flux $\dot{m}_a$ (kg/s) and thrust $T$ (kN). $\dot{m}_a \simeq 127.17$ kg/s, $T \simeq 123.8$ kN

![Diagram of a turbojet with nozzle and thrust calculations](image-url)
**Problem AB.3.12** The performance of a jet engine is limited by the capability of turbine-blade materials to withstand high temperatures. To investigate how a higher value of the postcombustion temperature can significantly improve the performance, consider the case of a turbojet flying at $M = 1.5$. The compressor pressure ratio is $P_{r_c} = 40$ and the adiabatic efficiencies (diffuser, compressor, turbine, and nozzle) are $\eta_d = 0.85$, $\eta_c = 0.8$, $\eta_t = 0.9$, and $\eta_n = 0.95$. In the calculations, assume that the nozzle is dimensioned to give $p_e = p_a$ and that the pressure loss in the combustor is negligible (i.e. $r_c = p_0_3/p_0_4 = 1$). Evaluate the results using $(\eta_b Q_R)/(c_p T_a) = 220$, $\gamma = 1.4$, and $a_a = 300$ m/s.

Consider first a turbojet designed to meet a peak temperature $T_{0_4}/T_a = 8$.

1. Determine the fuel-to-air mass ratio $f$.
2. Obtain the temperature jump across the turbine $T_{0_5}/T_{0_4}$.
3. Calculate the exit velocity $u_e$.
4. Determine the specific thrust $T/\dot{m}_a$ (in (kN · s)/kg) as well as the TSFC (in kg/(kN · s)).
5. Compute the propulsion efficiency $\eta_p$.

Consider now a turbojet with no limitation of peak temperature, so that one may use stoichiometric conditions in the combustor (i.e. $f = 0.067$).

6. Determine the peak temperature $T_{0_4}/T_a$.
7. Obtain the temperature jump across the turbine $T_{0_5}/T_{0_4}$.
8. Calculate the exit velocity $u_e$.
9. Determine the specific thrust $T/\dot{m}_a$ (in (kN · s)/kg) as well as the TSFC (in kg/(kN · s)).
10. Compute the propulsion efficiency $\eta_p$.

Comment on the results.
Problem AB.4.1

The figure below represents a schematic view of an ideal turbofan with compressor pressure ratio \( P_{rc} = 15 \) and fan pressure ratio \( P_{rf} = 2 \) designed to fly at \( M = 0.8 \). In the calculations, assume \( p_e = p_a \) and use \( a_a = 300 \text{ m/s} \) for the ambient sound speed, \( \gamma = 1.4 \) for the ratio of specific heats, \( T_{0a}/T_a = 8 \) for the peak temperature at the turbine inlet, and \( Q_R/(c_p T_a) = 200 \) for the heat of reaction.

1. Determine the fuel-to-air ratio \( f \). \( f \approx 0.02893 \)

2. Obtain the exhaust speed of the bypass stream \( u_{ef} \), expressing the result in the form \( u_{ef}/a_a \). \( u_{ef}/a_a \approx 1.369 \)

3. Compute the temperature ratio across the turbine \( T_{05}/T_{04} \) and the exhaust speed of the hot stream \( u_e/a_a \) for \( B = (4, 8, 12) \). \( T_{05}/T_{04} \approx (0.712, 0.588, 0.465) \), \( u_e/a_a \approx (3.481, 2.67, 1.497) \)

4. For those three values of the bypass ratio calculate the specific thrust \( T/\dot{m}_a \) and the TSFC. \( T/\dot{m}_a \approx (1.517, 1.95, 2.27) \text{ (kN \cdot s)/kg} \), TSFC\( \approx (0.019, 0.0148, 0.0127) \text{ kg/(kN \cdot s)} \)

Problem AB.4.2

Starting from

\[
\left( \frac{T}{\dot{m}_a} \right)_{max} = u \left[ (1 + f)\eta_{pr}\eta_g\eta_p \frac{\Delta h}{u^2} - \left( 1 - \frac{(1 + f)}{2} \right) \frac{\eta_m}{\eta_{pr}\eta_g\eta_p} \right],
\]

show that for a turboprop with optimized propeller-to-jet thrust ratio (optimum \( \alpha \)), the thrust is given by

\[
\frac{T}{\dot{m}_a} = M \sqrt{\gamma R T_a} \left[ \frac{(1 + f)\eta_{pr}\eta_g\eta_p}{(\gamma - 1)M^2} \frac{T_{05}}{T_{04}} \frac{T_{04}}{T_a} \left( 1 - \frac{1}{(p_{05}/p_{04})^{(\gamma-1)/\gamma}} \right) - \left( 1 - \frac{(1 + f)\eta_m}{2\eta_{pr}\eta_g\eta_p} \right) \right]
\]

Use the above expression, together with the approximation \( f \ll 1 \), to determine the compressor pressure ratio \( p_{rc} \) that provides the maximum thrust for a turboprop with ideal compressor turbine (\( \eta_t = 1 \)). The resulting value of \( p_{rc} \) should be a function of \( \gamma, M, T_{04}/T_a, r_c = p_{04}/p_{03}, \eta_d \) and \( \eta_c \).
Problem AB.4.3 The figure below represents a schematic view of a turbofan with small bypass ratio \( B = 3 \), similar to the Pratt & Whitney F119 developed for the F22 Raptor, including an afterburner downstream from the turbine. To determine the associated specific thrust \( \frac{T}{\dot{m}_a} \) at take off assume that the diffuser is ideal, so that \( p_{02} = p_a \) and \( T_{02} = T_a \). In the calculations, use \( a_a = 340 \text{ m/s} \) for the ambient sound speed and \( \gamma = 1.4 \) for the ratio of specific heats.

1. For a fan with pressure ratio \( p_{rf} = 1.5 \) and adiabatic efficiency \( \eta_f = 0.85 \), obtain the temperature ratio \( T_{08}/T_a \). \( T_{08}/T_a \simeq 1.145 \)

2. Assuming that the bypass flow is expanded isentropically in the fan nozzle to reach the ambient pressure at the exit, obtain the exit Mach number \( M_{ef} \) and the associated exit velocity \( u_{ef} \). \( M_{ef} \simeq 0.7836, u_{ef} \simeq 269 \text{ m/s} \)

3. For a compressor with pressure ratio \( p_{rc} = 15 \) and adiabatic efficiency \( \eta_c = 0.85 \), obtain the temperature ratio \( T_{03}/T_a \). \( T_{03}/T_a \simeq 2.374 \)

4. Using the condition \( P_{sc} + P_{sf} = P_{s1} \), compute the temperature ratio across the turbine \( T_{05}/T_{04} \). In the calculation, use \( T_{04}/T_a = 5 \) for the peak temperature at the turbine inlet. \( T_{05}/T_{04} \simeq 0.6385 \)

5. If the turbine adiabatic efficiency is \( \eta_t = 0.95 \), determine the pressure ratio \( p_{05}/p_a \). In the calculation, neglect pressure losses across the combustion chamber. \( p_{05}/p_a \simeq 2.805 \)

6. Consider that the combustion process in the afterburner increases the stagnation temperature by a factor of three (i.e., \( T_{06} = 3T_{05} \)) with a negligible pressure loss (i.e., \( p_{06} \simeq p_{05} \)). If the nozzle is ideal, determine the Mach number at the exit \( M_e \) and the associated jet speed \( u_e \). \( M_e \simeq 1.31, u_e \simeq 1,190 \text{ m/s} \)

7. Assuming a small fuel-to-air mass-flow ratio \( f \ll 1 \), calculate the specific thrust \( \frac{T}{\dot{m}_a} \) of the turbofan at take off. \( \frac{T}{\dot{m}_a} \simeq 1,997 \text{ (kN} \cdot \text{s)/kg} \)
Problem AB.4.4 The figure below represents schematically the engine of a VTOL aircraft, similar to the Pratt & Whitney F135-PW-600 turbofan engine used in the F-35B. Besides the compressor turbine, the engine has a separate power turbine. In flight mode, this power turbine is used to move the fan, so that the engine operates like a normal turbofan with $P_{r_c} = 20$, $P_{r_f} = 2$, and $B = 8$. For takeoff and landing, however, the engine switches to a turboshaft mode, with the power turbine used to move an external lift fan. The pressure ratio across the combustor is $r_c = \frac{p_{04}}{p_{03}} = 0.9$, while the adiabatic efficiencies are $\eta_d = 0.9$, $\eta_f = 0.85$, $\eta_c = 0.8$, $\eta_{ct} = 0.9$, $\eta_{pt} = 0.9$, $\eta_n = 0.97$, and $\eta_{fn} = 0.97$ for the diffuser, fan, compressor, compressor turbine, power turbine, nozzle, and fan nozzle, respectively. For the following analysis use $T_{04}/T_a = 7$, $(\eta_c Q_R)/(c_p T_a) = 200$, $a_a = 300$ m/s, and $\gamma = 1.3$.

- We begin by analyzing the performance of the engine in flight mode at $M = 0.8$:

1. Obtain the fuel-to-mass ratio $f$. $f \simeq 0.0235$

2. Compute the temperature behind the compressor turbine, giving the result in the form $T_{0n_a}/T_a$. In the calculation, use the condition $\mathcal{P}_{sc} = \mathcal{P}_{sc,t}$, stating that the compressor turbine is used to move the compressor. $T_{0n_a}/T_a \simeq 5.666$

3. Calculate the pressure behind the compressor turbine, giving the result in the form $p_{0n_a}/p_a$. $p_{0n_a}/p_a \simeq 9.2$

4. Determine the temperature and the pressure behind the fan, giving the result in the form $T_{0f}/T_a$ and $p_{0f}/p_a$, respectively. $T_{0f}/T_a \simeq 1.32$, $p_{0f}/p_a \simeq 2.864$

5. Assuming that $p_8 = p_a$, find the exhaust speed of the bypass stream $u_{ef}$, giving the result in the form $u_{ef}/a_a$. $u_{ef}/a_a \simeq 1.356$

6. Use the condition $\mathcal{P}_{sf} = \mathcal{P}_{spt}$ to obtain the temperature behind the power turbine, giving the result in the form $T_{0pt}/T_a$. $T_{0pt}/T_a \simeq 3.92$

7. Calculate the pressure jump across the power turbine, giving the result in the form $p_{0pt}/p_a$. $p_{0pt}/p_a \simeq 1.49$

8. Assuming that $p_6 = p_a$, obtain the exhaust speed of the hot stream $u_c$, giving the result in the form $u_c/a_a$. $u_c/a_a \simeq 1.4943$

9. Compute the specific thrust of the turbofan $\mathcal{T}/m_a$. $\mathcal{T}/m_a \simeq 1.5542$ (kN · s)/kg

- Consider now the performance of the engine when operating as a turboshaft during take-off/landing:

10. Compute $f$, $T_{0n_a}/T_a$, and $p_{0n_a}/p_a$ for $M = 0$. $f \simeq 0.0246$, $T_{0n_a}/T_a \simeq 5.785$, $p_{0n_a}/p_a \simeq 7.11$

11. Assuming $p_{0pt} = p_a$, determine the shaft power of the power turbine $\mathcal{P}_{spt}$ (which is used to move the lift fan), giving the result in the nondimensional form $\mathcal{P}_{spt}/(m_a a_a^2)$. $\mathcal{P}_{spt}/(m_a a_a^2) \simeq 6.4734$
**Problem AB.4.5** The figure below represents schematically a turbofan with bypass ratio $B = 6$ and compressor pressure ratio $P_{rc} = 18$ designed to fly at $M = 0.7$. The hot stream behind the turbine and the bypass stream behind the fan mix upstream from the nozzle, so that there is a single exhaust stream with velocity $u_e$. The pressure ratio across the combustor is $r_c = p_{04}/p_{03} = 0.9$, while the adiabatic efficiencies are $\eta_d = 0.9$, $\eta_f = 0.85$, $\eta_c = 0.8$, $\eta_t = 0.9$, and $\eta_n = 0.97$ for the diffuser, fan, compressor, turbine, and nozzle, respectively. For the following analysis use $T_{04}/T_a = 7$, $(\eta_bQ_R)/(c_pT_a) = 200$, $a_a = 300$ m/s, and $\gamma = 1.4$.

1. Obtain the fuel-to-mass ratio $f$. 

\[ f \approx 0.0214 \]

For the configuration selected, the condition that the stagnation pressure behind the turbine ($p_{05}$) equals the stagnation pressure behind the fan ($p_{07}$) must be used to determine the fan pressure ratio $P_{rf}$. In the computation, follow these steps:

2. Show that the condition $p_{05} = p_{07}$ can be written in the form $P_{rf} = (p_{05}/p_{04})r_cP_{rc}$.

3. Combine this last equation with the definition of the turbine efficiency to show that

\[
1 - \frac{T_{05}}{T_{04}} = \eta_t \left[ 1 - \left( \frac{P_{rf}}{r_cP_{rc}} \right)^{\gamma - 1} \right].
\]

4. Use the condition $P_{sf} + P_{sc} = P_{st}$ together with the last equation to derive the equation

\[
\frac{(T_{04}/T_a)\eta_t}{1 + \frac{\gamma - 1}{2}M^2} \left[ 1 - \left( \frac{P_{rf}}{r_cP_{rc}} \right)^{\gamma - 1} \right] = \frac{(P_{rc})^{\gamma - 1} - 1}{\eta_c} + B \frac{(P_{rf})^{\gamma - 1} - 1}{\eta_f},
\]

which can be used to solve for $P_{rf}$.

5. Determine $P_{rf}$. 

\[ P_{rf} \approx 1.68 \]

6. Compute $p_{05}/p_a = p_{07}/p_a$.  

\[ p_{05}/p_a \approx 2.26 \]

7. Calculate also $T_{05}/T_a$ and $T_{07}/T_a$.  

\[ T_{05}/T_a \approx 4.0, T_{07}/T_a \approx 1.304 \]

Assuming that mixing takes place at constant pressure (i.e. $p_{06} = p_{05} = p_{07}$) and with negligible heat losses:

8. Obtain the temperature immediately upstream from the nozzle, giving the result in the form $T_{06}/T_a$.  

\[ T_{06}/T_a \approx 1.69 \]

9. Find the exhaust speed $u_e$.  

\[ u_e \approx 391.53 \text{ m/s} \]

10. Determine the specific thrust $T/\dot{m}_a$ and the TSFC.  

\[ T/\dot{m}_a \approx 1.279 \text{ (kN \cdot s)/kg}, \text{ TSFC} \approx 0.0167 \text{ kg/(kN \cdot s)}\]
Problem AB.4.6 The figure below represents a turboshaft that is used on a small airplane flying at \( M = 0.5 \) at an altitude where \( T_a = 260 \text{ K} \). Since the gas is expanded through the power turbine to a pressure close to the ambient value \( (p_6 \simeq p_a) \), the resulting exhaust jet has a small speed \( u_e \ll u \) that does not contribute significantly to the thrust. To obtain the specific thrust \( \frac{T}{\dot{m}_a} \) and the TSFC of the turboshaft follow the steps suggested below. In the calculations, use \( \gamma = 1.4 \) and \( R = 287 \text{ J/(kg K)} \).

1. Assuming that the diffuser has an efficiency \( \eta_d = 0.98 \), determine the stagnation values of the temperature and pressure at the compressor inlet, giving the results in the form \( p_{02}/p_a \) and \( T_{02}/T_a \). \( p_{02}/p_a \simeq 1.182 \), \( T_{02}/T_a \simeq 1.05 \).

2. For a compressor with pressure ratio \( p_{rc} = 15 \) and adiabatic efficiency \( \eta_c = 0.85 \), obtain the temperature ratio \( T_{03}/T_{02} \). \( T_{03}/T_{02} \simeq 2.374 \).

3. Assuming that the maximum temperature at the turbine inlet is \( T_{04} = 1500 \text{ K} \), use the energy balance across the combustion chamber to determine the value of the fuel-to-air ratio \( f \). In the calculation, employ \( Q_R = 45 \times 10^6 \text{ J/kg} \) and \( \eta_b = 1 \). \( f \simeq 0.0196 \).

4. Using the condition \( P_{sc} = P_{st} \), determine the value of the temperature ratio across the compressor turbine \( T_{05}/T_{04} \). \( T_{05}/T_{04} \simeq 0.75 \).

5. If the compressor turbine adiabatic efficiency is \( \eta_t = 0.95 \), determine the pressure ratio \( p_{05}/p_{04} \). \( p_{05}/p_{04} \simeq 0.343 \).

6. Analyze the expansion across the power turbine to determine the associated shaft power \( P_{pt} \), giving the result in the dimensionless form \( P_{pt}/(\dot{m}_a u^2) \). In the computation, neglect pressure losses across the combustion chamber (i.e., \( p_4 = p_{04} \)) and use \( \eta_{pt} = 0.96 \) for the adiabatic efficiency of the power turbine. \( P_{pt}/(\dot{m}_a u^2) \simeq 17.07 \).

7. Taking into account that \( u_e \ll u \), calculate the specific thrust \( \frac{T}{\dot{m}_a} \) and TSFC for the turboshaft. The efficiencies of the propeller and the gear box are \( \eta_{pr} = 0.8 \) and \( \eta_g = 0.93 \), respectively. \( \frac{T}{\dot{m}_a} \simeq 1.89 \text{ (kN} \cdot \text{s)/kg}, \text{ TSFC} \simeq 0.010366 \text{ kg/(kN} \cdot \text{s)} \).
Problem AB.4.7 A turboprop with peak temperature $T_{04} = 1400$ K and compressor pressure ratio $P_{rc} = 12$ is to be used for a large cargo airplane with cruise conditions $M = 0.6$ and $T_a = 220$ K. For the calculations below use $\gamma = 1.4$, $\eta_bQ_R/(c_pT_a) = 200$, $a_a = 297$ m/s, and $p_e = p_a$.

- Obtain the fuel-air ratio $f$ assuming that the adiabatic efficiency of the compressor is $\eta_c = 0.85$. $f \simeq 0.0206$

- Calculate the temperature ratio $T_{05}/T_{04}$ and the pressure ratio $p_{05}/p_{04}$ across the high-pressure (compressor) turbine, whose adiabatic efficiency is $\eta_t = 0.85$. $T_{05}/T_{04} \simeq 0.795$, $p_{05}/p_{04} \simeq 0.38$

- Obtain the ideal enthalpy drop available downstream from the turbine $\Delta h = h_{0a} - h_{7s}$, giving the result in the form $\Delta h/(c_pT_a)$ with

$$\frac{\Delta h}{c_pT_a} = \frac{T_{05}}{T_a} \left( 1 - \frac{1}{(p_{05}/p_a)^{(\gamma-1)/\gamma}} \right).$$

Use $r_c = 1$ and $\eta_d = 0.9$ for the pressure ratio across the combustor and for the adiabatic efficiency of the diffuser, respectively. $\Delta h/(c_pT_a) \simeq 1.98$

- Using $\eta_n = 0.98$ and $\eta_p\eta_g\eta_{pt} = 0.4$ determine the optimal value $\alpha_{opt}$ of the fraction $\alpha$ of available enthalpy to be used in the power turbine. $\alpha_{opt} \simeq 0.78$

- For this value $\alpha = \alpha_{opt}$ calculate the specific thrust $T/\dot{m}_a$ and the TSFC of the turboprop. $T/\dot{m}_a \simeq 1.045$ (kN · s)/kg, TSFC $\simeq 0.0197$ kg/(kN · s)

- Compare the resulting values of $T/\dot{m}_a$ and TSFC with those corresponding to a turbojet with the same efficiencies and the same values of $T_{04}/T_a$ and $P_{rc}$. Comment on the results. $T/\dot{m}_a \simeq 0.766$ (kN · s)/kg, TSFC $\simeq 0.027$ kg/(kN · s)
Problem AB.4.8 The Bristol Siddeley BS100 engine was a British turbofan with small bypass ratio $B = 1.2$ designed and built over fifty years ago for the Hawker Siddeley P.1154, a supersonic vertical/short take-off and landing (V/STOL) fighter aircraft. For thrust augmentation at takeoff the engine used “plenum chamber burning” (PCB), an alternative to the standard afterburner in which the reheat is applied in the bypass stream. To explore the advantages of this alternative, consider the two configurations represented below. In the analysis, assume that the nozzles have variable geometry to give an exit pressure equal to the ambient pressure. The left-hand-side figure corresponds to a standard afterburner, for which

$$\frac{T}{\dot{m}_a} = (1 + f + f^*)u_e^* + Bu_{ef}$$

at takeoff, whereas the figure on the right corresponds to the PCB engine, for which

$$\frac{T}{\dot{m}_a} = (1 + f)u_e + (B + f^*)u_{ef}$$

at takeoff. In the above expressions $f^* = \dot{m}_f^*/\dot{m}_a$, with $\dot{m}_f^*$ and $\dot{m}_a$ representing, respectively, the reheat fuel burning rate and the air mass flux through the core engine. The compressor and fan pressure ratios are $P_{rc} = 16$ and $P_{rf} = 2$, and the adiabatic efficiencies are $\eta_f = 0.85$, $\eta_c = 0.8$, $\eta_t = 0.9$, $\eta_n = 0.97$, and $\eta_{fn} = 0.97$ for the fan, compressor, turbine, nozzle, and fan nozzle, respectively. For the following analysis use $T_0 = 1600 \text{ K}$, $(\eta_b Q_R)/(c_p T_a) = 200$, $a_a = 340 \text{ m/s}$, $T_a = 288 \text{ K}$, and $\gamma = 1.3$ and neglect all pressure losses associated with combustion. The maximum combustion temperature for the afterburner and the PCB is $T_{0\tau^*} = T_{0\tau} = 2500 \text{ K}$.

1. Compute the fuel-to-air mass ratio $f$ in the main combustor.
2. Determine the temperature ratio across the turbine $T_{0\tau}/T_0$.
3. Calculate the temperature behind the fan $T_{0\tau}$, giving the result in the form $T_{0\tau}/T_a$.

For the afterburner engine:
4. Obtain the exhaust speed for the bypass stream $u_{ef}$.
5. Compute the value of $f^*$.
6. Determine the exhaust speed for the core engine $u_e^*$.

For the PCB engine:
7. Obtain the exhaust speed for the core engine $u_e$.
8. Compute the value of $f^* = \dot{m}_f^*/\dot{m}_a$.
9. Determine the exhaust speed for the bypass stream $u_{ef}^*$.

Using the above values,
10. Calculate the specific thrust (in (kN · s)/kg) at takeoff for both designs and comment on the results.