MAE 210B – FLUID MECHANICS II – SPRING 2017

HOMEWORK ASSIGNMENT # 1 (Due on April 17, 2017)

Problem 1: In cylindrical coordinates, $\mathbf{v} = v_r \mathbf{e}_r + v_\theta \mathbf{e}_\theta + v_z \mathbf{e}_z$. Use the expressions

$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \frac{\partial}{\partial z} \right)$$

and

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

to derive the three components of the momentum equation

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}.$$

In the derivation, note that

$$\frac{\partial \mathbf{e}_r}{\partial \theta} = \mathbf{e}_\theta \qquad \text{and} \qquad \frac{\partial \mathbf{e}_\theta}{\partial \theta} = -\mathbf{e}_r.$$

Problem 2: For axisymmetric flow $\mathbf{v} = v_r(r,z)\mathbf{e}_r + v_z(r,z)\mathbf{e}_z$ show that

$$\boldsymbol{\omega} = \nabla \wedge \mathbf{v} = \omega_\theta \mathbf{e}_\theta = \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right) \mathbf{e}_\theta.$$

If the flow is inviscid, show that the vorticity equation reduces to

$$\frac{D}{Dt}\left(\frac{\omega_\theta}{r}\right) = \frac{\partial}{\partial t}\left(\frac{\omega_\theta}{r}\right) + v_r \frac{\partial}{\partial r}\left(\frac{\omega_\theta}{r}\right) + v_z \frac{\partial}{\partial z}\left(\frac{\omega_\theta}{r}\right) = 0.$$

Problem 3: Consider the planar inviscid flow of a constant density fluid through a contraction in a channel. The velocity profile upstream from the contraction (i.e. as $x \to -\infty$ for $0 \le y \le H_1$) is given by $\mathbf{v} = U_1(y)\mathbf{e}_x = Ay\mathbf{e}_x$ and the pressure is $p = p_1$, where A and p_1 are constant. Obtain the velocity profile $\mathbf{v} = U_2(y)\mathbf{e}_x$ and the pressure $p = p_2$ downstream from the contraction (i.e. as $x \to +\infty$ for $0 \le y \le H_2$).