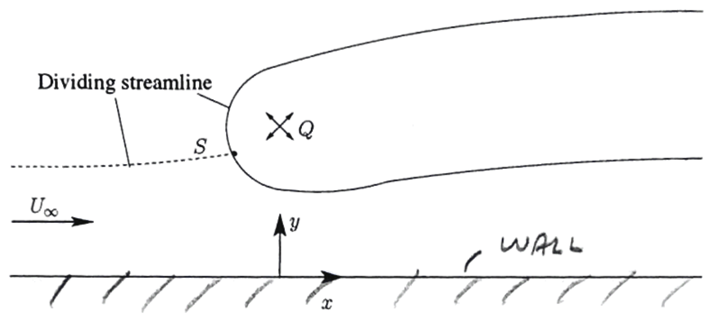
Use a simple 2D potential problem to investigate the flow in the jet-engine intake of a Harrier aircraft flying at cruise speed U_∞. The presence of the intake lip is modeled as a pair of mirror sources of intensity $Q = (\pi/2)U_\infty h$ placed at $y = \pm hi$. Compute the complex potential $f(z)$ and use it to determine the velocity distribution, the location of the stagnation points, the streamline bounding the flow going into the jet engine, and the associated intake volume rate (volume of air that enters the engine per unit time). Obtain also the velocity $V_W(x)$ and pressure $p_W(x)$, along the side of the aircraft ~~upstream from the intake~~ $(y = 0)$ ~~...~~

ON THE WALL



$$f = U_\infty z + \frac{Q}{2\pi}\left[\ln(z-hi) + \ln(z+hi)\right] = U_\infty z + \frac{Q}{2\pi}\ln(z^2+h^2) = U_\infty h\left[\frac{z}{h} + \frac{1}{4}\ln\left(\left(\frac{z}{h}\right)^2+1\right)\right]$$

$$\frac{df}{dz} = U_\infty + \frac{Q}{2\pi}\left(\frac{1}{z-hi} + \frac{1}{z+hi}\right) = U_\infty + \frac{Q}{2\pi}\left[\frac{2z}{z^2+h^2}\right] = u - iv$$

$$u = U_\infty + \frac{Q}{\pi}\left[\frac{x\left[x^2-y^2+h^2\right] + 2xy^2}{(x^2-y^2+h^2)^2+4x^2y^2}\right]$$

$$v = \frac{Q}{\pi}\,\frac{y(x^2-y^2+h^2) - 2x^2y}{(x^2-y^2+h^2)^2+4x^2y^2}$$

STAGNATION POINTS $\frac{df}{dz} = 0$ $\rightarrow$ $\boxed{\frac{z}{h} = -\frac{1}{4} \pm \frac{\sqrt{15}}{4}i}$

$$f = \phi + i\psi \rightarrow \frac{\psi}{U_\infty h} = \frac{y}{h} + \frac{1}{4}\left[\arctan\left(\frac{2\frac{x}{h}\frac{y}{h}}{\left(\frac{x}{h}\right)^2 - \left(\frac{y}{h}\right)^2+1}\right)\right]$$

STREAMLINES $\psi = \text{CONSTANT}$.

THE DIVIDING STREAMLINE CROSSES $\frac{x}{h} = -\frac{1}{4}$, $\frac{y}{h} = \frac{\sqrt{15}}{4}$

$$\frac{y}{h} + \frac{1}{4}\arctan\left[\frac{2\,x/h\;y/h}{\left(\frac{x}{h}\right)^2 - \left(\frac{y}{h}\right)^2+1}\right] = \frac{\sqrt{15}}{4} - \frac{1}{4}\arctan(\sqrt{15})$$

AS $x \rightarrow -\infty$ $\quad y/h = \frac{\sqrt{15}}{4} - \frac{1}{4}\arctan(\sqrt{15}) \rightarrow$ INTAKE VOLUME RATE

$$\boxed{U_\infty h\left(\frac{\sqrt{15}}{4} - \frac{1}{4}\arctan(\sqrt{15})\right)}$$

AT $z = x \rightarrow v = 0$, $u = U_\infty + \frac{Q}{\pi}\frac{x}{x^2+h^2} = U_\infty\left(1 + \frac{1}{2}\frac{x/h}{(x/h)^2+1}\right) = V_W(x)$

$$p_W - p_\infty = \frac{1}{2}\rho U_\infty^2\left[1 - \left(1 + \frac{1}{2}\frac{x/h}{(x/h)^2+1}\right)^2\right]$$

WITH CIRCULATION

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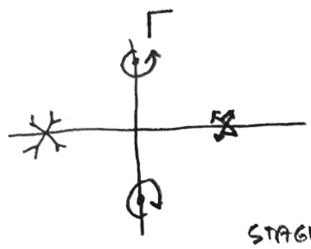
Consider the flow induced by the superposition of a source of strength Q located at $(a, 0)$, a sink of strength $-Q$ located at $(-a, 0)$, a vortex line ~~of strength~~ Γ located at $(0, ai)$, and a vortex line ~~of strength~~ $-\Gamma$ located at $(0, -ai)$. Find the complex potential and the stagnation points. Represent schematically the streamlines, considering separately the cases $Q/\Gamma > 1$, $Q/\Gamma = 1$, and $Q/\Gamma < 1$.



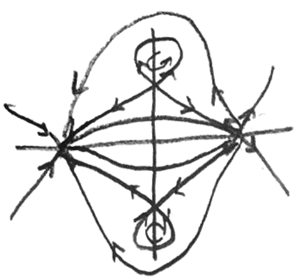
$$f(z) = \frac{Q}{2\pi} \ln\left(\frac{z-a}{z+a}\right) + \frac{\Gamma i}{2\pi} \ln\left(\frac{z+ai}{z-ai}\right)$$

$$\frac{df}{dz} = \frac{Q}{\pi}\frac{a}{z^2-a^2} + \frac{\Gamma a}{\pi}\frac{1}{z^2+a^2}$$

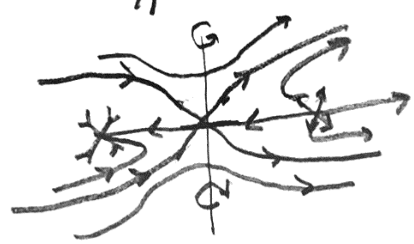
STAGNATION POINTS:

$$u - iv = 0 = \frac{\Gamma}{\pi}\,\frac{z^2-a^2+Q/\Gamma\,(z^2+a^2)}{z^4-a^4} = 0 \Rightarrow \frac{z}{a} = \sqrt{\frac{1-Q/\Gamma}{1+Q/\Gamma}}$$

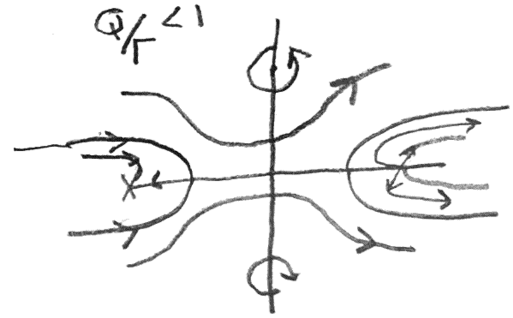
$Q/\Gamma > 1$



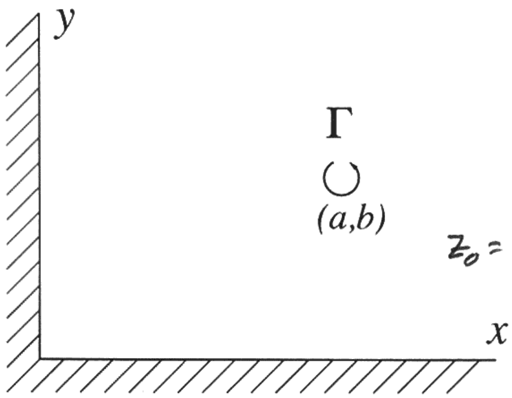
$Q/\Gamma = 1$



$Q/\Gamma < 1$



Consider the flow induced by a vortex line with positive circulation Γ located at $z = a + bi$ in the vicinity of a corner. Find the complex potential and the stagnation points. Represent schematically the streamlines, considering separately the cases $b/a > 1$, $b/a = 1$, and $b/a < 1$.

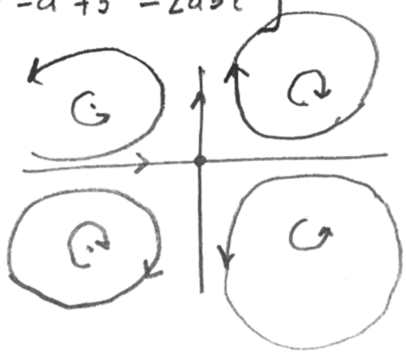


$z_0 = a + bi$

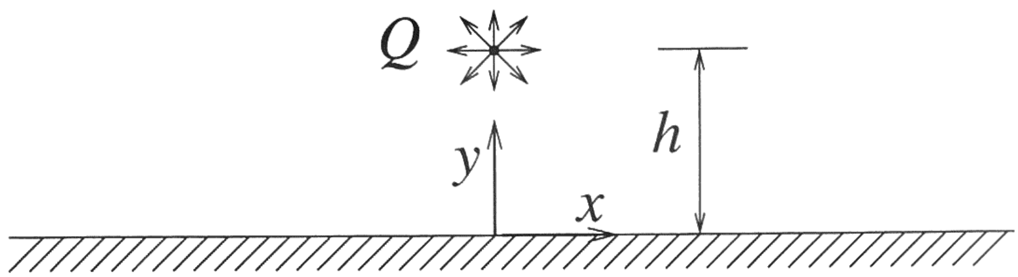
$$f(z) = \frac{\Gamma i}{2\pi}\Big[-\ln[z-(a+bi)] + \ln[z-a+bi] + \ln(z+a-bi] - \ln[z+a+bi]\Big]$$

$$= \frac{\Gamma i}{2\pi} \ln\left[\frac{z^2-a^2+b^2+2abi}{z^2-a^2+b^2-2abi}\right]$$

$$\frac{df}{dz} = 0 \rightarrow z = 0$$



A source of strength Q is located at a distance h from a flat wall. Determine the complex potential and use it to compute the streamlines, the velocity distribution along the wall $v_w(x)$, the accompanying pressure distribution $p_w(x)$, and the force associated with the presence of the source $-\int_{-\infty}^{+\infty}(p_w - p_\infty)\mathrm{d}x$.



$$f(z) = \frac{Q}{2\pi}\ln(z-hi) + \frac{Q}{2\pi}\ln(z+hi) = \phi + i\psi = \frac{Q}{2\pi}\ln\left(z^2+h^2\right)$$

$$\psi = \frac{Q}{2\pi}\arctan\left(\frac{2xy}{x^2-y^2+h^2}\right)$$ → STREAMLINES ψ = CONSTANT

$$\frac{df}{dz} = \frac{Q}{2\pi(z-hi)} + \frac{Q}{2\pi}\frac{1}{z+hi} = U - iV,$$ ALONG THE WALL $z = x \rightarrow V = 0$, $U = \boxed{V_w = \frac{Q}{\pi}\frac{x}{x^2+h^2}}$

BERNOULLI'S EQ: $P - P_\infty = -\frac{1}{2}\rho(U^2+V^2) \rightarrow \boxed{-(P_w - P_\infty) = \frac{1}{2}\rho\left(\frac{Q}{\pi}\right)^2\frac{x^2}{(x^2+h^2)^2}}$

$$-\int_{-\infty}^{+\infty}(P_w - P_\infty)\,dx = \frac{1}{2}\rho\left(\frac{Q}{\pi}\right)^2\int_{-\infty}^{\infty}\frac{x^2\,dx}{(x^2+h^2)^2} = \boxed{\frac{1}{4\pi}\frac{\rho}{h}Q^2}$$