Problem 1: For axisymmetric flow in spherical coordinates \( \mathbf{v} = v_r(r, \theta) \mathbf{e}_r + v_\theta(r, \theta) \mathbf{e}_\theta \). Show that
\[
\omega = \nabla \times \mathbf{v} = \omega_\phi \mathbf{e}_\phi = \frac{1}{r} \left( \frac{\partial (rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right) \mathbf{e}_\phi.
\]
If the flow is inviscid, show that the vorticity equation reduces to
\[
\frac{D}{Dt} \left( \frac{\omega_\phi}{r \sin \theta} \right) = \frac{\partial}{\partial t} \left( \frac{\omega_\phi}{r \sin \theta} \right) + v_r \frac{\partial}{\partial r} \left( \frac{\omega_\phi}{r \sin \theta} \right) + \frac{v_\theta}{r} \frac{\partial}{\partial \theta} \left( \frac{\omega_\phi}{r \sin \theta} \right) = 0.
\]
In the derivation, you might want to use the geometrical expressions \( \partial \mathbf{e}_r / \partial \phi = \sin \theta \mathbf{e}_\phi \) and \( \partial \mathbf{e}_\theta / \partial \phi = \cos \theta \mathbf{e}_\phi \).

Problem 2: To estimate the force acting on a sewer lid located in a ditch of depth \( h \), consider the planar configuration sketched in the figure below, where the origin of the coordinate system is located at the center of the lid. In the analysis, follow these steps:

1. Determine the potential motion resulting from the superposition of a uniform flow of velocity \( U_\infty \), a sink of strength \(-Q\) located at \( z = -a \) and a source of strength \( Q \) located at \( z = +a \), computing the complex potential as well as the stagnation points as a function of \( \Lambda = Q / (\pi U_\infty a) \).

2. Sketch the resulting stream lines for \( \Lambda > 1 \), \( \Lambda = 1 \) and \( \Lambda < 1 \).

3. Verify that, for this last case \( \Lambda < 1 \), the flow field can be associated with the problem at hand, writing the two relationships that link \( \Lambda \) and \( l/a \) with \( h/l \).

4. Obtain the pressure distribution along the horizontal axis \( z = x \), giving the result in the form \( (p - p_\infty) / (\rho U_\infty^2 / 2) \) as a function of \( \Lambda \) and \( x/a \).

5. Compute the force acting on the sewer lid, making use of the results derived above in 3) to write the result in the form \( F / (\rho U_\infty^2 l / 2) \) as a function of \( \Lambda \).

6. Calculate the dimensionless force \( F / (\rho U_\infty^2 l / 2) \) for the particular case \( h/l = 3\pi / 8 \).
Problem 3: To investigate the so-called ground effect on the flow field around an aircraft moving parallel to the ground at height $h$ use the potential flow sketched in the figure below, with the effect of the aircraft assumed to be that of a vortex of circulation $-\Gamma$. Obtain the complex potential and the associated stagnation points as a function of the parameter $K = \Gamma/((\pi U_\infty h)$, plotting the resulting stream lines when $K > 1$, $K = 1$, and $K < 1$. 

\[ \begin{align*} 
    \text{Stream lines} & \quad \text{Vortex} \\
    \text{Upstream} & \quad -\Gamma \\
    \text{Downstream} & \quad \text{Stagnation point} \\
    \text{Ground} & \quad h \\
    \text{Axis} & \quad x, y \\
\end{align*} \]