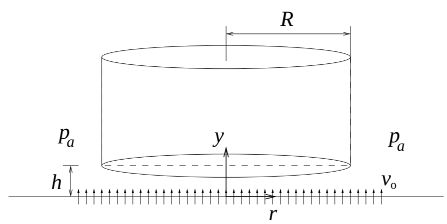
MAE 210B – FLUID MECHANICS II – SPRING 2017

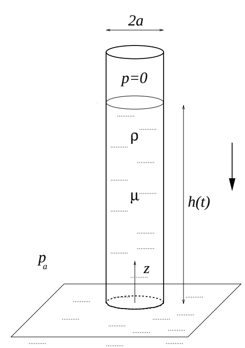
HOMEWORK ASSIGNMENT # 5 (Due on May 26, 2017)

Problem 1: Consider a solid cylinder of mass M and radius R located over a planar porous surface as indicated in the figure below. Gas is injected normal to the surface with velocity v_o, inducing overpressures $p - p_a$ that allow the cylinder to levitate at a small distance $h \ll R$. Give the criterion needed for the flow to be dominated by viscosity. Obtain the pressure distribution $p(r) - p_a$ as well as the force acting on the cylinder and the height h.



Problem 2: The bottom end of an empty vertical tube of radius a, closed at the top, is put in contact with a pool of oil. Because of the ambient overpressure, the liquid begins to flow into the tube, forming a column of increasing height $h(t)$ whose evolution in time is to be investigated assuming that the motion is dominated by viscosity. In particular:

- Obtain the value of the height h_∞ corresponding to the equilibrium position, reached asymptotically for large times.
- Give the condition that determines whether the motion is dominated by viscosity.
- Obtain the evolution of $h(t)$ as well as the pressure distribution along the pipe $p(x, t)$ for $0 < x < h$.
- Compute the force acting on the pipe as a function of time $\bar{F} = F_z(t)\bar{e}_z$.



Problem 3: A plate of length L is initially sitting on a horizontal plane surrounded by a stagnant atmosphere at pressure p_a. At a given instant, we begin to rotate the plate with constant angular velocity $\Omega = \mathrm{d}\alpha/\mathrm{d}t$ by applying a given torque M at its left end, as sketched in the figure. For the analysis, use the approximation $h(x, t) = x \tan(\alpha) \simeq x\alpha$, valid for $\alpha \ll 1$.

1. Demonstrate that for values of α sufficiently smaller than a critical value, to be determined, the fluid motion in the gap formed between the plate and the wall is dominated by viscosity.
2. Obtain the velocity profile v_x in the gap as a function of the unknown value of $P_l(x, t) = -\partial p/\partial x$ as well as the associated volume flux at a given section $Q = \int_0^{\alpha x} v_x \mathrm{d}y$.
3. Using continuity, write an equation linking Q and Ω, and integrate it to compute the pressure distribution $p(x, t)$ (in the integration, you may anticipate that $x^3 P_l \to 0$ as $x \to 0$).
4. Determine the torque $M(t)$ needed to provide a constant angular velocity Ω.