Problem 1: To investigate the flow near the contact point of windshield wiper with inclination angle $\alpha$ and relative velocity $U$ consider the planar configuration shown in the figure. For the description we shall use a reference frame moving with the windshield wiper, as indicated in the figure.

1. Estimate the size of the region around the contact point where the motion is dominated by viscous forces.

2. Formulate the problem in terms of a stream function $\psi(r, \theta)$, defined such that $rv_r = (\partial \psi)/(\partial \theta)$ and $v_\theta = -(\partial \psi)/(\partial r)$.

3. Use the II theorem to show that $\psi/(Ur) = f(\theta, \alpha)$. Determine the function $f(\theta, \alpha)$.

4. Calculate the velocity components $v_r(\theta)$ and $v_\theta(\theta)$, as well as the pressure distribution $p(r, \theta) - p_\infty$.

5. Obtain the force acting on the windshield wiper

Problem 2: A spherical gas bubble of radius $a$ moves with relative velocity $U$ through a liquid of density $\rho$ and viscosity $\mu$. Introduce a stream function $\psi$ defined by $r^2 \sin(\theta)v_r = (\partial \psi)/(\partial \theta)$ and $r \sin(\theta)v_\theta = -(\partial \psi)/(\partial r)$ to study the motion of the liquid around the bubble, assuming that $\rho Ua/\mu \ll 1$ and that the viscosity of the gas is negligibly small. In the computation use the spherical coordinates indicated in the figure. Determine the velocity and the pressure in the liquid as well as the drag force acting on the bubble.
Problem 3: Consider the motion induced in a liquid of density $\rho$ and viscosity $\mu$ by a solid sphere rotating with constant angular speed $\Omega$. In the description, use the spherical coordinates indicated in the figure. Give the condition for the flow to be dominated by viscosity. Assuming that $v_r = v_\theta = 0$, determine the velocity $v_\phi(r, \theta)$ and the pressure $p - p_\infty$. Obtain the torque that the fluid exerts on the sphere.