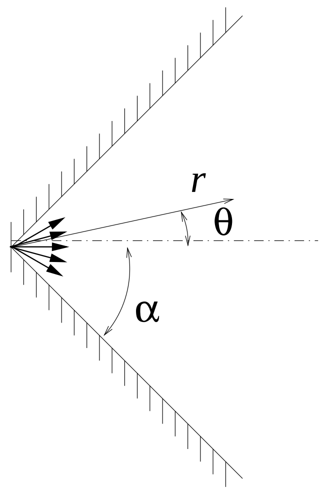
# MAE 210B – FLUID MECHANICS II – SPRING 2017

## HOMEWORK ASSIGNMENT # 6 (Due on June 9, 2017)

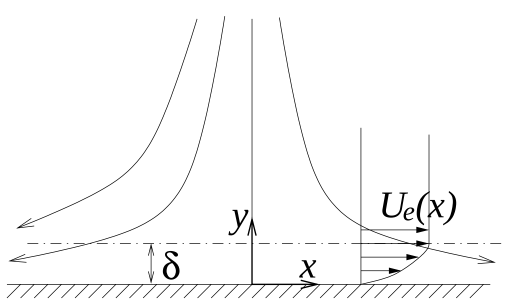
**Problem 1:** Analyze the flow induced between two non-parallel plane walls by a line source (or sink) located at their intersection point, as indicated in the figure. The character of the resulting flow depends on the volume flux per unit distance perpendicular to the plane of the flow $Q$ (note that the dimensions of $Q$ are area per unit time).

1. Give the condition needed for the fluid motion to be dominated by viscous forces.
2. Formulate the problem in terms of a stream function $\psi(r,\theta)$, defined such that $rv_r = (\partial\psi)/(\partial\theta)$ and $v_\theta = -(\partial\psi)/(\partial r)$.
3. Use the $\Pi$ theorem to show that $\psi/Q = f(\theta,\alpha)$ and determine the function $f(\theta,\alpha)$.
4. Calculate the velocity components $v_r(\theta)$ and $v_\theta(\theta)$, as well as the pressure distribution $p(r,\theta) - p_\infty$.




**Problem 2:** The potential velocity field near a stagnation point is given by $v_x = Ax$ and $v_y = -Ay$ in terms of the constant strain rate $A$, with corresponding slip velocity on the wall $U_e(x) = Ax$. Investigate the boundary layer that forms on the wall following the two alternative approaches delineated below.

1. Write the boundary-layer equations along with the boundary conditions that determine the velocity in the boundary layer $u(x,y)$ and $v(x,y)$. Use an order-of-magnitude analysis to determine the characteristic values of the boundary layer thickness $\delta_c$ and of the velocity components in the boundary layer $u_c$ and $v_c$. Show that the problem admits a self-similar solution in terms of the similarity variables $U(\eta) = u/u_c$ and $V(\eta) = v/v_c$, where $\eta = y/\delta_c$. Write the equations with boundary conditions for the self-similar problem.
2. Assume the velocity profile $u/U_e = 1 - e^{-y/\delta(x)}$ and determine $\delta^*$, $\theta$, and $\tau_w$. Use the momentum integral equation to write a differential equation for the evolution of $\delta(x)$. Integrate using the condition that $\delta$ must be finite at $x = 0$.



**Problem 3:** Consider a semi-infinite porous plate aligned with a free stream of uniform velocity $U_\infty$. A constant suction velocity $v_w = -v_o$ exists on the plate surface, so that far from the leading edge the boundary layer approaches a solution independent of $x$, as shown in the figure below.

- Estimate the thickness of the downstream boundary layer $\delta$ as well as the characteristic value of the distance from the leading edge $L$ at which the far-field solution is reached.
- Determine the velocity profile $v_x(y)$ for $x \gg L$, and use it to compute the associated values of $\delta^*$, $\theta$, and $\tau_w$.
- Use Karman's integral equation to study the development of the boundary layer at distances $x \sim L$ from the leading edge. Assume that the values of $H = \delta^*/\theta$ and $T = \tau_w/(\mu U_\infty/\theta)$ in this region remain constant and equal to those of the far-field solution. Give the solution for $\theta/\delta$ as a function of $x/L$, and investigate the limiting solutions arising for $x/L \ll 1$ and $x/L \gg 1$.