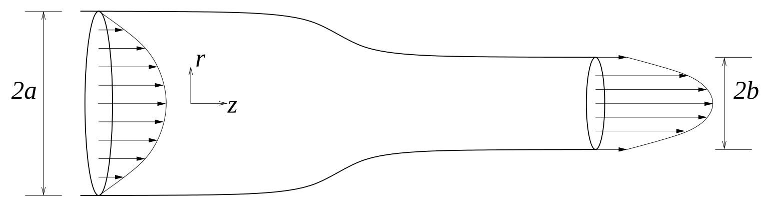
MAE 210B – FLUID MECHANICS II – SPRING 2017

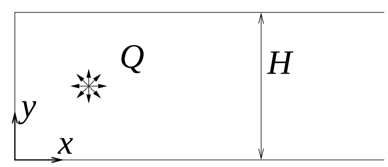
1st MIDTERM EXAM Assigned: **5/3 8:00AM** Due: **5/5 8:00AM**

Problem 1: Consider the steady axisymmetric flow of a fluid of constant density ρ and constant viscosity μ in the contraction shown in the figure, which connects two coaxial cylindrical pipes of radii a and $b < a$. Upstream from the contraction the velocity is given by the Poiseuille profile $v_z = U[1 - (r/a)^2]$, where U is the maximum velocity. Assuming that $\rho U a/\mu \sim \rho U b/\mu \gg 1$:

- Write the equations with boundary conditions that determine the velocity and pressure fields.
- Show that the vorticity magnitude $\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}$ is given by $\omega_\theta = 2Ur/a^2$ everywhere in the flow field.
- Obtain the velocity profile $v_z(r)$ downstream from the contraction, as well as the total pressure drop across the contraction $p_{z\to-\infty} - p_{z\to+\infty}$.



Problem 2: A point source of strength Q is located at $(x, y) = (H/2, H/2)$ inside a semiinfinite channel of thickness H, as indicated in the figure. Determine the complex potential $f(z)$ as well as the pressure along the vertical wall $p(0, y)$ for $0 \le y \le H$.



Problem 3: A uniform stream of velocity U_∞ flows past a two-dimensional symmetric body of length $2L$ and height $2H$. An approximate solution can be generated by superimposing on the free stream a source of strength Q located at $z = -a$ and a sink of strength $-Q$ located at $z = a$, as sketched in the figure. To analyze the flow, follow these steps:

1. Write the complex potential.
2. Obtain the stagnation points and use the resulting equation to relate the relative semi-length L/a with the parameter $\Lambda = Q/(\pi U_\infty a)$.
3. Determine the stream function as well as the equation for the stream line that defines the contour of the body.
4. Write an equation relating the relative semi-height H/a with the parameter Λ.
5. For $H/a = 2$, obtain the corresponding values of Λ and L/a.
6. For this last case, compute the maximum velocity over the body, achieved at $z = \pm Hi$.