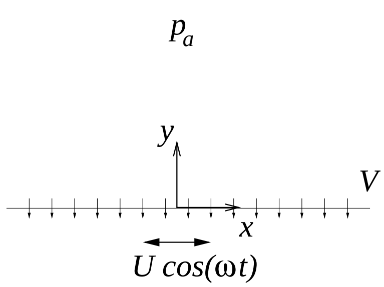
MAE 210B – FLUID MECHANICS II – SPRING 2017

2nd MIDTERM EXAM Assigned: **5/31 8:00AM** Due: **6/2 8:00AM**

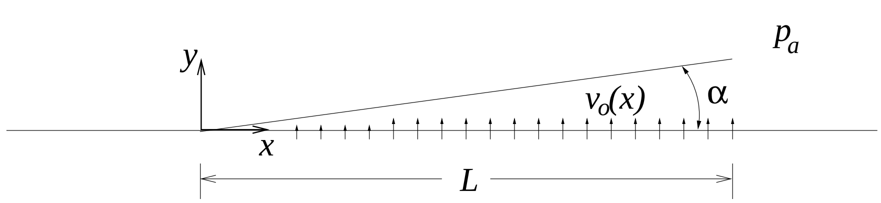
Problem 1: Consider the modified Stokes problem defined in the figure below. The oscillating wall, moving parallel to itself with velocity $U\cos(\omega t)$, is a porous surface through which the fluid is entrained with constant normal velocity $-V$.

1. Anticipating that the x component of the velocity $u(y,t)$ is only a function of the distance to the wall y and the time t, obtain the distribution of transverse velocity $v(x,y,t)$.
2. Assuming that the pressure takes the uniform constant value $p = p_a$ far from the wall, compute the pressure distribution $p(x,y,t)$.
3. Write the equation with boundary conditions that determine $u(y,t)$.
4. Consider the limit $V \gg (\nu\omega)^{1/2}$, demonstrating that the motion is quasi-steady. Find the associated solution for $u(y,t)$.
5. Consider the limit $V \ll (\nu\omega)^{1/2}$, demonstrating that the effect of convection is negligible in the first approximation. Find the associated solution for $u(y,t)$.
6. In the general case, $V \sim (\nu\omega)^{1/2}$ write the problem in dimensionless form with use made of the scales U, ω^{-1}, and ν/V for u, t, and y, respectively. Obtain the exact solution for u/U, verifying that the limiting solutions determined above are recovered in the corresponding limits $V \gg (\nu\omega)^{1/2}$ and $V \ll (\nu\omega)^{1/2}$, respectively.



Problem 2: A fluid of density ρ and viscosity μ is injected with normal velocity $v_o(x)$ into a channel of length L bounded by two flat walls forming an angle $\alpha \ll 1$, as indicated in the figure. For the steady flow established in the channel:

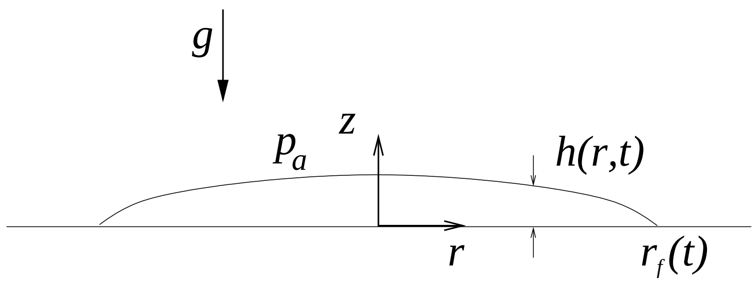
1. Give the condition for the motion to be dominated by viscous forces[1].
2. Obtain the pressure distribution along the channel $p(x) - p_a$.
3. For the particular case $v_o = V(x/L)^2$, where V is a constant, obtain the force acting on the upper wall of the channel as well as the moment exerted with respect to the left end $x = 0$.



[1]The condition you derive should involve only the known quantities ρ, μ, L, and α along with the characteristic value v_c of $v_o(x)$

Problem 3: An **axisymmetric** liquid pool of kinematic viscosity ν spreads over a flat horizontal surface under the action of gravity. Initially the pool has radius r_o and thickness distribution $h_o(r)$, such that $V = \int_0^{r_o} 2\pi h_o r \mathrm{d}r$ is the total volume of liquid.

1. Give the condition for the motion to be dominated by viscous forces[2].
2. Write the equation with initial and boundary conditions that determine the thickness distribution $h(r,t)$ as well as the radial location of the pool edge $r_f(t)$.
3. Rewrite the problem in dimensionless form by introduction of the scales r_o, V/r_o^2, and $(\nu/g)(r_o^8/V^3)$ for r, h, and t, respectively.
4. Consider the evolution for $t \gg (\nu/g)(r_o^8/V^3)$. Obtain the characteristic values $h_\infty(t) \ll V/r_o^2$ and $r_\infty(t) \gg r_o$ of h and r_f in this limit.
5. Determine the self-similar solution for the long-time evolution by rewriting the problem in terms of the rescaled variables $H(\eta) = h/h_\infty(t)$ and $\eta = r/r_\infty(t)$. Obtain in particular an expression for the pool-edge location $r_f(t)$ for $t \gg (\nu/g)(r_o^8/V^3)$.



[2]The condition you derive should involve only the known quantities ν, g, V, and r_o