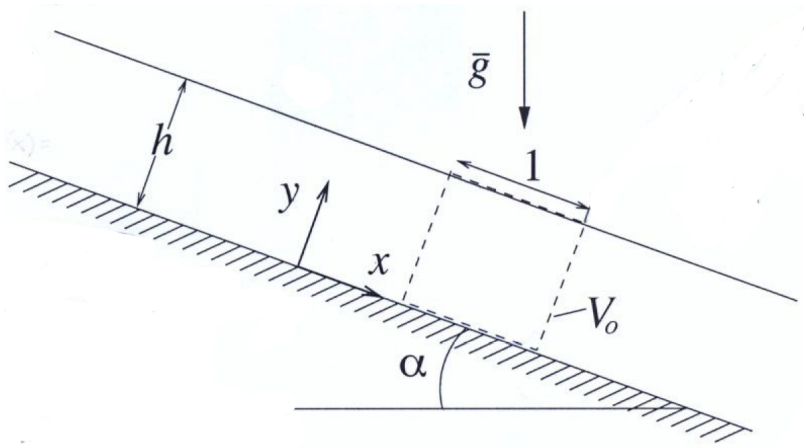
Problem 4



Consider the steady planar motion of a perfect liquid of density ρ, viscosity μ, thermal conductivity k, and specific heat c that flows down over an inclined plane of angle α due to the action of gravity, forming a layer of constant thickness h. To solve the problem, use the coordinate system (x, y) indicated in the figure, in which gravity can be expressed as $\vec{g} = g \sin\alpha \vec{e}_x - g \cos\alpha \vec{e}_y$. To analyze the resulting liquid flow, assume that the velocity components $v_x(y)$ and $v_y(y)$ are a function of the distance to the wall, and that the air surrounding the liquid has a density $\rho_a \ll \rho$ and a viscosity $\mu_a \ll \mu$, such that in the first approximation it moves without significant pressure differences, exerting on the liquid a negligible viscous force.

(i) Determine the velocity component $v_y(y)$.

(ii) Obtain the pressure field $p(x, y)$.

(iii) Compute $v_x(y)$.

(iv, BONUS) Calculate the force per unit surface area, $-p\vec{n} + \bar{\bar{\tau}} \cdot \vec{n}$, that the liquid exerts on the wall.