Problem #1: A porous sphere of radius \( a \) is immersed in a fluid of density \( \rho \). The fluid is entrained with radial velocity \( v_r = -V_0 \) through the porous surface of the sphere. For the buoyancy-free, steady sphericosymmetrical motion induced in the surrounding fluid,

1. Obtain the velocity \( v_r(r) \)

2. Compute the trajectory of a fluid particle initially located at \( r = r_i > a \), as well as the time \( t_o \) at which the fluid particle reaches the sphere surface.

3. Determine the pressure distribution \( p - p_\infty \), where \( p_\infty \) is the pressure value far from the sphere.

Problem #2: An infinitely long porous cylinder of radius \( R \) is surrounded by a liquid of density \( \rho \) and viscosity \( \mu \). The cylinder moves axially with velocity \( v_z = U \) and entrains liquid through its surface pores with a suction velocity \( v_r = -V \), inducing for \( r > R \) an axisymmetric velocity field with components \( v_r(r), v_z(r) \) and \( v_\theta = 0 \). At large radial distances \( r \gg R \) from the cylinder the liquid is at rest with uniform pressure \( p_\infty \). Assume in the following analysis that the effect of gravity on the fluid motion is negligible.

1. Use the continuity equation to determine the radial velocity component \( v_r(r) \) for \( R \leq r < \infty \).

2. From the radial component of the momentum equation compute the pressure distribution, demonstrating in particular that \( \partial p / \partial z = 0 \).

3. Determine the axial velocity distribution \( v_z(r) \) as a function of the suction Reynolds number \( Re = \rho V R / \mu \).

4. Calculate the trajectory of the fluid particle located initially at \( r = r_o \) and \( z = 0 \).
MAE 201 – MECHANICS OF FLUID – FALL 2018

HOMEWORK ASSIGNMENT # 3 (Due at 5PM on Oct 23, 2018)

Problem #3: A liquid of density $\rho$ and viscosity $\mu$ occupies the gap formed between two porous coaxial cylinders of radius $R_i$ and $R_o$ that are rotating with angular velocities $\Omega_i$ and $\Omega_o$, respectively. The liquid is injected through the surface of the inner cylinder with normal velocity component $v_r = \dot{V}_i$ and is removed through the outer surface. The resulting steady, planar, axially symmetric flow field is described in terms of $v_r(r)$, $v_\theta(r)$, and $p(r)$.

1. Determine the radial velocity component $v_r(r)$. Obtain in particular its value on the surface of the outer cylinder $v_r(R_o)$.

2. Obtain the azimuthal velocity component $v_\theta(r)$. To that end, write the $\theta$-component of the momentum equation in terms of the parameter $Re = \rho \dot{V}_i R_i / \mu$ and look for solutions of the form $v_\theta \propto r^\alpha$, demonstrating that the general solution can be expressed as $v_\theta = C_1/r + C_2 r^{Re+1}$. Obtain the values of the integration constants $C_1$ and $C_2$.

3. Calculate $p(r) - p_i$, the difference of the pressure $p(r)$ from its inner-surface value $p_i = p(R_i)$.

\[ 1) \ \nabla \vec{V} = 0 \implies \nabla V_r = \frac{R_i \dot{V}_i}{R_o} \quad v_r(R_o) = \frac{R_i \dot{V}_i}{R_o} \]

\[ 2) \ \frac{\partial V_r}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) = \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) \right) - Re \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) + \frac{v_\theta}{r^2} \right) = 0 \]

\[ \Rightarrow \quad \frac{v_\theta}{r^2} = \frac{C_1}{r} + C_2 r^{Re+1} \]

\[ \begin{align*}
V_\theta(R_i) &= \Omega_i R_i = \frac{C_1}{R_i} + C_2 R_i^{Re+1} \\
V_\theta(R_o) &= \Omega_o R_o = \frac{C_1}{R_o} + C_2 R_o^{Re+1} \\
C_1 &= (R_i R_o)^2 \Omega_i R_o - \Omega_o R_i \\
C_2 &= \frac{\Omega_o R_o - \Omega_i R_i}{R_0^{Re+2} - R_i^{Re+2}} \\
\end{align*} \]

\[ 3) \ \frac{dp}{dr} = -\frac{\sigma}{2} \left( \frac{v_r dr}{dr} - \frac{v_r^2}{r} \right) \quad \Rightarrow \quad p - p_i = \frac{(R_i \dot{V}_i)^2 + C_1^2}{2} \left( \frac{1}{R_i^{Re+2}} - \frac{1}{r^{Re+2}} \right) + \frac{C_2}{2R_i^{Re+2}} \left( \frac{v_r^2 R_i^{Re+2} - R_i^{2Re+2}}{R_i^{Re+2} - R_i} \right) + \frac{\Omega_i C_2}{R_i} \left( \frac{R_i^{Re+2} - R_i^{2Re+2}}{R_i^{Re+2} - R_i} \right) \]