A nuclear missile is designed to fly at supersonic speeds (velocities $U$ larger than the sound velocity $c$) at high altitudes. In the solution that appears, a bow shock is present at a standoff distance $d$ from the missile tip. To characterize the solution and determine the drag force $F$ acting on the missile, a series of experiments is conducted in a supersonic wind tunnel using a small experimental model geometrically similar to the real prototype. Analyze the results following these steps:

1. Use the II theorem to reduce the parametric dependence of the drag $F$ and standoff distance $d$. To set up the problem, note that the compressibility of the gas is characterized by the ambient sound speed $c$, which should be considered at the start of the analysis, along with the ambient values of the density and viscosity of the gas $\rho$ and $\mu$. Since the density changes across the shock, gravity might also influence the solution through the resulting buoyant force.

2. Derive the conditions under which both gravity and viscosity do not influence the solution. The reasoning should be substantiated with an order-of-magnitude analysis of the momentum equation to show clearly which dimensionless parameters measure the relative importance of buoyant and viscous forces (relative to the convective acceleration).

3. The experimental tests are carried out at normal ambient conditions ($\rho_e = 1.22 \text{ kg/m}^3$, $\mu_e = 1.98 \times 10^{-5} \text{ kg/(m s)}$, $c_e = 340 \text{ m/s}$) with a model of diameter $D_e = 0.01 \text{ m}$, significantly smaller than that of the real missile ($D_m = 1 \text{ m}$), giving the results shown in the table. Use the results, together with the assumption that $g$ and $\mu$ have a negligible influence, to deduce the variation with flight speed $U_m$ of the drag force $F_m$ and standoff distance $d_m$ corresponding to the missile flying at cruise altitude, where $\rho_m = 0.36 \text{ kg/m}^3$, $\mu_m = 1.44 \times 10^{-5} \text{ kg/(m s)}$, and $c_m = 295 \text{ m/s}$.

4. Check that the criteria needed to neglect $g$ and $\mu$ are satisfied both in the experiment and for the real missile conditions.

5. Determine a simplified expression for the drag force when the Mach number is very large (i.e., $U/c \gg 1$).

<table>
<thead>
<tr>
<th>$U_e$ (m/s)</th>
<th>$F_e$ (N)</th>
<th>$d_e$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>9.2</td>
<td>0.012</td>
</tr>
<tr>
<td>700</td>
<td>37.50</td>
<td>0.01</td>
</tr>
<tr>
<td>1000</td>
<td>86.00</td>
<td>0.005</td>
</tr>
<tr>
<td>2000</td>
<td>383</td>
<td>0</td>
</tr>
<tr>
<td>4000</td>
<td>1532</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{1) } F_d &= \frac{F}{d^2} = \frac{\rho U^2}{\mu} \left( \frac{D}{u} \right) \\
\text{2) } \frac{\frac{\rho U^2}{\mu}}{\frac{\rho U^2}{\mu}} &= \frac{\rho U^2}{\mu} \\
\text{3) } \frac{\rho U^2}{\mu} &= \frac{\rho U^2}{\mu} \\
\end{align*}
\]

\[
\begin{align*}
\text{1) } d &= d_0 \left( \frac{U}{c} \right) \left( \frac{D}{u} \right) \\
\text{2) } \frac{\frac{\rho U^2}{\mu}}{\frac{\rho U^2}{\mu}} &= \frac{\rho U^2}{\mu} \\
\text{3) } \frac{\rho U^2}{\mu} &= \frac{\rho U^2}{\mu} \\
\end{align*}
\]

\[
\begin{align*}
\text{1) } \frac{F_m}{d_m} &= \frac{F_e}{d_e} \left( \frac{D_m}{D_e} \right)^2 \\
\text{2) } \frac{\rho U^2}{\mu} &= \frac{\rho U^2}{\mu} \\
\text{3) } \frac{\rho U^2}{\mu} &= \frac{\rho U^2}{\mu} \\
\end{align*}
\]

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\begin{align*}
\text{1) } \frac{F_m}{d_m} &= \frac{F_e}{d_e} \left( \frac{D_m}{D_e} \right)^2 \\
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\text{3) } \frac{\rho U^2}{\mu} &= \frac{\rho U^2}{\mu} \\
\end{align*}
\]
Problem #2: Consider a semi-infinite fluid domain bounded by a horizontal wall that remains at rest for \( t < 0 \). At \( t = 0 \), the wall is suddenly set in parallel motion with velocity \( V = at \), where \( a \) is the constant acceleration. The motion induced in the fluid is parallel to the wall. The resulting velocity \( u(y, t) \) is only a function of time and of the distance to the wall \( y \). Write the equation with initial and boundary conditions that determine \( u(y, t) \). Rewrite the problem in dimensionless form showing that the solution is self-similar.

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \quad t > 0 \\
\begin{align*}
\gamma = 0: & \quad u = At \\
\gamma = \infty: & \quad u = 0
\end{align*} \Rightarrow \frac{u_x}{\epsilon} \sim \nu \frac{u_x}{\epsilon^2} \Rightarrow x = \sqrt{\nu \tau}
\]

\[
\bar{u} = \frac{u}{At}, \quad \eta = \frac{y}{\sqrt{\nu t}}
\]

\[
\frac{d\bar{u}}{d\tau} = A \left( \bar{u} \bar{\eta} \right) + \frac{1}{2} \frac{d\bar{u}}{d\eta}, \quad \frac{d^2\bar{u}}{d\eta^2} = \frac{A}{\nu} \frac{d^2\bar{u}}{d\eta^2} + \gamma \frac{d\bar{u}}{d\eta} - \bar{u} = 0 \quad \gamma = 0: \quad \bar{u} = 1 \quad \gamma = \infty: \quad \bar{u} = 0
\]

\[
\bar{u} = \frac{\gamma^{1/2}}{\sqrt{2\pi}} \exp\left(-\eta^2/2\right) \Rightarrow \text{Parabolic Cylinder Function}
\]