Problem 1

When do the celsius & fahrenheit temp scales converge.

Let $x$ be celsius & $y$ fahrenheit

\[
\frac{9}{5}x + 32 = y, \quad x = y \implies \frac{9}{5}x + 32 = x \implies 32 = \frac{4}{5}x \implies x = y = -40
\]

\[
K = ^\circ C + 273.15 = 233.15K
\]

\[
\circ R = ^\circ F + 459.67 = 419.67 \circ R
\]
Problem 2

KNOWN: A closed system undergoes a cycle consisting of three processes.

FIND: Determine the net work of the cycle and the heat transfer for process 2-3. Identify whether the cycle is a power cycle or a refrigeration cycle and explain.

SCHEMATIC AND GIVEN DATA:

**State 2**

- $V_2 = 0.2 \text{ m}^3$
- $U_2 = U_1 + 100 \text{ kJ}$

**State 3**

- $P_3 = P_1 = 1 \text{ bar}$
- $V_3 = V_2 = 0.2 \text{ m}^3$

**State 1**

- $P_1 = 1 \text{ bar}$
- $V_1 = 2 \text{ m}^3$

Process 2 $\rightarrow$ 3
- constant volume

Process 1 $\rightarrow$ 2
- $pV = \text{constant}$
- $\Delta U_{12} = 100 \text{ kJ}$

$p-V$ Diagram

ENGINEERING MODEL:
1. The gas is a closed system.
2. Kinetic and potential energy effects are negligible.
3. Process 1-2 is polytropic in which $pV = \text{constant}$.
4. Process 2-3 is constant volume.
5. Process 3-1 is constant pressure and adiabatic.

ANALYSIS:
Cycle work is the sum of work associated with each process in the cycle

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{31}$$

Process 1-2 is a polytropic process with $pV = \text{constant}$. Therefore, $p = \text{constant}/V$. 
Problem 2 Continued

\[ W_{12} = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} \frac{\text{(constant)} \, dV}{V} = \text{(constant)} \int_{V_1}^{V_2} \frac{dV}{V} = \text{(constant)} \ln \left( \frac{V_2}{V_1} \right) = p_i V_1 \ln \left( \frac{V_2}{V_1} \right) \]

\[ W_{23} = (1 \text{ bar})(2 \text{ m}^3) \left( \frac{10^5 \text{ N}}{\text{m}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) \ln \left( \frac{0.2 \text{ m}^3}{2 \text{ m}^3} \right) = -460.5 \text{ kJ} \]

Process 2-3 is constant volume; thus \( W_{23} = \int_{V_2}^{V_3} p \, dV = 0 \text{ kJ} \)

Process 3-1 is constant pressure; thus \( W'_{31} = \int_{V_3}^{V_1} pdV = p_i (V_1 - V_3) \)

\[ W'_{31} = (1 \text{ bar})(2 \text{ m}^3 - 0.2 \text{ m}^3) \left( \frac{10^5 \text{ N}}{\text{m}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 180 \text{ kJ} \]

Substituting the work associated with each process yields the cycle work

\[ W_{\text{cycle}} = (-460.5 \text{ kJ}) + 0 \text{ kJ} + 180 \text{ kJ} = -280.5 \text{ kJ} \]

Since the net cycle work is into the cycle (negative), the cycle is a \textit{refrigeration cycle}.

For process 2-3, an energy balance is

\[ \Delta KE_{23} + \Delta PE_{23} + (U_3 - U_2) = Q_{23} - W_{23} \]

Neglecting changes in kinetic energy (\( \Delta KE_{23} = 0 \)) and potential energy (\( \Delta PE_{23} = 0 \)), substituting \( W_{23} = 0 \) (determined above), and solving for \( Q_{23} \) yield

\[ Q_{23} = U_3 - U_2 \]

For the cycle,

\[ (U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0 \]

Solving for \( U_3 - U_2 \) yields

\[ (U_3 - U_2) = -(U_2 - U_1) - (U_1 - U_3) \]

From the problem statement, \( (U_2 - U_1) = 100 \text{ kJ} \)

For process 3-1, an energy balance is

\[ \Delta KE_{31} + \Delta PE_{31} + (U_1 - U_3) = Q_{31} - W_{31} \]
Problem 2 Continued

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$), substituting $Q_{31} = 0$ since process 3-1 is adiabatic, and solving for $(U_1 - U_3)$ give

$$ (U_1 - U_3) = -W_{31} = -180 \text{ kJ} $$

Substituting for changes in internal energy gives

$$ (U_3 - U_2) = -(100 \text{ kJ}) - (-180 \text{ kJ}) = 80 \text{ kJ} $$

Solving for $Q_{23}$

$$ Q_{23} = U_3 - U_2 = 80 \text{ kJ} $$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.

4. As an alternative solution, for the overall cycle, $Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = W_{cycle}$

Thus, $Q_{23} = W_{cycle} - Q_{12} - Q_{31}$

For process 1-2, an energy balance is

$$ \Delta KE_{12} + \Delta PE_{12} + \Delta U_{12} = Q_{12} - W_{12} $$

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$) and solving for heat transfer give

$$ Q_{12} = \Delta U_{12} + W_{12} = 100 \text{ kJ} + (-460.5 \text{ kJ}) = -360.5 \text{ kJ} $$

The heat transfer is negative during process 1-2, denoting energy transfer by heat from the gas during this process.

Since process 3-1 is adiabatic, $Q_{31} = 0 \text{ kJ}$.

Substituting values for $W_{cycle}$ and heat transfer associated with each process yields

$$ Q_{23} = (-280.5 \text{ kJ}) - (-360.5 \text{ kJ}) - 0 \text{ kJ} = 80 \text{ kJ} $$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.
Problem 3

KNOWN: Data are provided for helium contained in a closed rigid tank fitted with an electrical resistance.

FIND: Plot the change in energy of the helium, w+w, for 220 and comment.

SCHEMATIC & GIVEN DATA:

![Schematic of helium tank with electric resistor]

ENGR. MODEL:
1. The helium is the system.
2. For the system, W = 0.

ANALYSIS: An energy rate balance reads
\[
\frac{dE}{dt} = Q - W \frac{Q}{W_{\text{min}}}
\]

As the system receives energy by heat transfer from the resistor at a rate of 1 kW and loses energy by heat transfer to its surroundings at the rate of 5 kW,

\[
Q = [1000 - 5t] \text{ W}
\]

Thus,

\[
\frac{dE}{dt} = 1000 - 5t
\]

And

\[
\Delta E = \int_0^t \frac{dE}{dt} dt = \int_0^t (1000 - 5t) dt = \left[1000t - \frac{5t^2}{2}\right] \text{ W.min}
\]

\[
\Delta E = \left[1000t - \frac{5t^2}{2}\right] \text{ W.min} \frac{1 \text{kJ}}{10^3 \text{W}} \frac{60 \text{s}}{1 \text{min}} = \left[50t - 0.15t^2\right] \text{kJ}
\]

\[
\Delta E = E(t) - E(0) = E(t) - E_0
\]

- 0 to 200 min: Energy increases from its initial value at t = 0: E_0.
- 200 to 400 min: Energy decreases to its initial value: E_0.
- 400 min: Energy decreases from its initial value: E_0.

Note that since any arbitrary value E_0 can be assigned to the energy of the system at t = 0, no particular significance can be attached to the value of energy at the initial state or at any other state. Only changes in the energy of the system have significance.
**Problem 4**

**Find Q & W**

<table>
<thead>
<tr>
<th>State</th>
<th>( P ) (kN/m(^2))</th>
<th>( V ) (m(^3)/kg)</th>
<th>( U ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>690</td>
<td>0.3</td>
<td>2643</td>
</tr>
<tr>
<td>2</td>
<td>276</td>
<td>0.7</td>
<td>2615</td>
</tr>
</tbody>
</table>

**Assumptions:**
1) polytropic process
2) negligible PE & KE
3) steam is the closed system

**Analysis:**

\[
W = \int_{1}^{2} pdV = \int_{1}^{2} \frac{P}{V^n} dV = \left( \frac{P_2 V_2^n - P_1 V_1^n}{1-n} \right)
\]

Now we need to find \( n \):

\[
P_1 V_1^n = P_2 V_2^n \Rightarrow \frac{P_1}{P_2} = \left( \frac{V_2}{V_1} \right)^n \Rightarrow n = \frac{\ln \left( \frac{P_1}{P_2} \right)}{\ln \left( \frac{V_2}{V_1} \right)} = \frac{\ln \left( \frac{690 \, \text{kN/m}^2}{276 \, \text{kN/m}^2} \right)}{\ln \left( \frac{0.7 \, \text{m}^3/\text{kg}}{0.3 \, \text{m}^3/\text{kg}} \right)} = 1.0467
\]

\[
W = \frac{(276 \, \text{kN/m}^2)(0.7 \, \text{m}^3/\text{kg}) - (690 \, \text{kN/m}^2)(0.3 \, \text{m}^3/\text{kg})}{1 - 1.0467} = 295 \, \text{kJ/kg} \text{ (out)}
\]

Heat transfer can now be found using first law:

\[
\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow Q = \Delta U + W = \frac{Q}{m} = \Delta U + \frac{W}{m}
\]

\[
\frac{Q}{m} = (2615 \, \text{kJ/kg} - 2643 \, \text{kJ/kg}) + 295 \, \text{kJ/kg} = 267 \, \text{kJ/kg (in)}
\]
8) \( P_1 = 1 \text{ bar} = 100 \frac{kN}{m^2} \), \( P_2 = 500 \frac{kN}{m^2} \)

\( T_1 = 298.15 \) \( K \)

\( v_1 = 1 \text{ m}^3/\text{kg} \)

\( C_p = 1 \frac{kJ}{kg K} \), \( R = \frac{8.314}{28} \frac{kJ}{kg K} \)

\( c_v = C_p - R \)

\( c_v = 0.7031 \frac{kJ}{kg K} \)

Final volume, \( v_2 = \frac{v_1 P_1}{P_2} \)

\( v_2 = 0.2 \text{ m}^3/\text{kg} \).

Let "i" be the intermediate state.

9) \( T_i = T_1 \frac{v_2}{v_1} = 69.63 \text{ K} \)

Process 1-\( i \) :- \( q = \Delta h = C_p \Delta T = 1 \times (69.63 - 298.15) \)

\( = -238.52 \frac{kJ}{kg K} \)

\( \Delta u = \Delta h - P \Delta V = -158.52 \frac{kJ}{kg K} \)

Process 1-2 :- \( q = \Delta u = c_v \Delta T = 0.7031 \times (298.15 - 69.63) \)

\( = 167.70 \frac{kJ}{kg K} \)

\( \Delta h = \Delta u + v \Delta P = 247.7 \frac{kJ}{kg K} \)
\[ q = -238.52 + 167.70 = -70.82 \ \text{kJ/kg K} \]

\[ \Delta u = -158.52 + 167.70 = 9.18 \ \text{kJ/kg K} \]

\[ \Delta h = -238.52 + 247.7 = 9.18 \ \text{kJ/kg K} \]

\[ w = q - \Delta u = -80 \ \text{kJ/kg K} \]

b) \[ \Delta u = \Delta h \text{ in both cases, since } \Delta h = \Delta u + \Delta (pv) \]
\[ \Delta h = \Delta u + \Delta (RT) \]
\[ \text{Same initial and final temperature} \]

\[ T_1 = \frac{T_1 P_2}{P_1} = 1490.75 \text{ K} \]

Process 1-1:
\[ q = \Delta u = C_v \Delta T = 0.7031 \times (1490.75 - 298.15) \]
\[ = 838.1 \ \text{kJ/kg K} \]

Process 1-2:
\[ q = \Delta h = c_p \Delta T = 1 \times (298.15 - 1490.75) \]
\[ = -1192.6 \ \text{kJ/kg K} \]

\[ q = -384.5 \ \text{kJ/kg K} \]

\[ w = q - \Delta u = -363.68 \ \text{kJ/kg K} \]
6) **1st law**, \( \delta q = \delta u + pdV \)

**Ideal gas**, \( \Rightarrow \delta u = c_v dT \), \( p = \frac{RT}{V} \)

\[ \delta q = c_v dT + \frac{RT}{V} dV \]

\[ \frac{\delta q}{T} = c_v \frac{dT}{T} + R \frac{dV}{V} \]

\[
\int_{T_1}^{T_2} \frac{\delta q}{T} = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{V_1}^{V_2} \frac{dV}{V}
\]

\[
\int_{V_1}^{V_2} \frac{\delta q}{T} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}
\]
7) \[ \frac{PV}{RT} = 1 + \frac{B(T)}{V} + \frac{C(T)}{V^2}. \quad \text{Isothermal} \]

Work done per unit mass \( \int_{V_1}^{V_2} \) = \[ \int_{V_1}^{V_2} p \, dv \]

\[ = \frac{1}{RT} \int_{V_1}^{V_2} dV \left[ \frac{1}{V} + \frac{B}{V^2} + \frac{C}{V^3} \right] \]

\[ = \frac{1}{RT} \left[ \ln \frac{V_2}{V_1} + \frac{B(V_2-V_1)}{V_1V_2} + \frac{C(V_2^2-V_1^2)}{2V_1^2V_2} \right] \]