1. A semi-infinite space $x > 0$ with thermal diffusivity $\alpha$ is bounded by a flat wall $x = 0$. Initially the temperature everywhere is $\theta = \theta_\infty$. A constant heat flux is applied at the wall at the initial instant of time, resulting in the problem

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} \quad \left\{ \begin{array}{l} \theta(x, 0) = \theta_\infty \\ \frac{\partial \theta}{\partial x}(0, t) = -\beta, \quad \theta(\infty, t) = \theta_\infty \end{array} \right.$$ 

where $\beta$ is a constant (the heat flux divided by the thermal conductivity). Show that the temperature evolution for $t > 0$ is self-similar and reduce the solution to the integration of a second-order boundary-value problem.

2. Consider a metal rod of length $L$ with initial temperature $\theta_i(x)$ at $t = 0$. The left end of the rod is kept at a constant temperature $\theta = \theta_l$ while the right end is insulated, so that $\partial \theta / \partial x = 0$ at $x = L$. Obtain the evolution of the temperature $\theta(x, t)$ for $t > 0$.

3. Find the solution $u(x, y)$ to Poisson’s equation $\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = xy$ inside the square $0 < x < a$, $0 < y < b$ with $u$ vanishing on the boundary. In solving the problem, it is convenient to express the dependent variable in the form $u(x, y) = v(x, y) + u_p(x, y)$, where $u_p$ is a particular integral (i.e. such that $\partial^2 u_p / \partial x^2 + \partial^2 u_p / \partial y^2 = xy$), while $v(x, y)$ correspondingly satisfies $\partial^2 v / \partial x^2 + \partial^2 v / \partial y^2 = 0$ (subject to non-homogeneous boundary conditions, to be determined).

4. Solve Laplace’s equation for $u$ inside the semicircle $r < a$, $0 < \theta < \pi$ with boundary conditions $u = 0$ on $\theta = 0$ and $\theta = \pi$ for $0 \leq r \leq a$ and $u = \theta(\pi - \theta)$ on $r = a$ for $0 \leq \theta \leq \pi$. 