1. Solve the equation
\[ y' = (\cosh x)\sqrt{y} - e^x y, \quad y(0) = 1. \]

2. Solve the inhomogeneous initial-value problem
\[ y'' + \omega^2 y = xe^{-x^2}, \quad y(0) = 0, \quad y'(0) = v \]
using a Green’s function approach.

3. Show that the origin is a regular singular point of the differential equation
\[ xy'' + (\gamma + 1 - x)y' + \beta y = 0. \]
Take \( \gamma > -1 \) and non-integer. Solve the indicial equation and find the recurrence relation. Show that there are polynomial solutions for integer \( \beta \), and obtain the polynomials explicitly for \( \beta = 0, 1 \) and 2.

4. Show that the solution to the equation
\[ y'' - \frac{yy'}{x} = 0 \]
with boundary conditions \( y(1) = 0 \) and \( y'(1) = C \) is
\[ x = \exp \left( \int_0^1 \frac{d\zeta}{\zeta^2/2 + \zeta + C} \right). \]
Obtain the solution explicitly for \( C = 1/2 \).

5. Find the most general solution \( u(x, y) \) to the following equation, consistent with the boundary condition stated:
\[ \frac{\partial u}{\partial x} + (x + y) \frac{\partial u}{\partial y} = 0, \quad u(0, y) = \ln y. \]

6. Find the solution to
\[ \frac{3}{4} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = 0 \]
with boundary conditions \( u = e^{x^2} \) and \( \partial u/\partial y = x^3 \) along the line \( y = 0 \).