**P1:** Consider a buoyancy-free, steady planar motion occurring in a semi-infinite space filled with a perfect liquid of density $\rho$, viscosity $\mu$, thermal conductivity $k$, and specific heat $c$. The liquid is bounded by an infinitely long porous wall that moves parallel to itself with velocity $U_w$ while removing the liquid with a normal suction velocity $V_w$. Far from the wall, the horizontal velocity component is zero and the pressure is $p = p_\infty$. The ambient value of the temperature $T = T_\infty$ is higher than the value $T_w$ at the wall.

1. Determine the velocity components $v_x(y)$ and $v_y(y)$ and well as the pressure field $p(x, y)$.

2. Obtain the temperature distribution $T(y)$.

3. Compute the viscous force acting on the wall per unit surface $\tau_w e_x = \vec{t}' \cdot \vec{n}_w$, where $\vec{n}_w$ is the unit vector normal to the wall.

4. Determine the heat flux from the fluid to the wall $q_w = k(\nabla T) \cdot \vec{n}_w$.

5. For an alternative computation procedure for $\tau_w$ and $q_w$, apply the momentum and energy conservation equations to the control volume indicated in the figure. In dealing with the balance equations, evaluate explicitly the contribution of each term, stating clearly the reasons justifying any simplifications introduced.

**P2:** A sphere of radius $a$ is immersed in a liquid of density $\rho$, viscosity $\mu$, thermal conductivity $k$, and specific heat $c$. The liquid is sucked with radial velocity $v_r = -V_o$ through the porous surface of the sphere. For the buoyancy-free, steady spherically symmetric motion induced in the surrounding liquid,

1. Obtain the velocity $v_r(r)$

2. Compute the trajectory of a fluid particle initially located at $r = r_i > a$, as well as the time $t_i$ at which the fluid particle reaches the sphere surface.

3. Determine the pressure distribution $p - p_\infty$, where $p_\infty$ is the pressure value far from the sphere.

4. Calculate the temperature distribution in the liquid $T(r)$ assuming that the temperature of the sphere is $T_s$, smaller then the value $T_\infty$ found in the surrounding ambient atmosphere.

5. Obtain the amount of heat transferred to the sphere per unit time.
**Problem 3**: A liquid of density $\rho$, viscosity $\mu$, specific heat $c$, and thermal conductivity $k$ fills the annular gap formed between two concentric cylinders of radii $R_i$ and $R_o$, respectively. The inner cylinder moves with axial velocity $U$ while the outer cylinder rotates around its axis with angular velocity $\Omega$, inducing in the liquid an axially symmetric motion described in cylindrical coordinates by the velocity components $v_r(r)$, $v_z(r)$, and $v_\theta(r)$ with associated pressure distribution $p(r)$.

1. Obtain the radial velocity $v_r(r)$.

2. Calculate the axial velocity $v_z(r)$.

3. Determine the azimuthal velocity $v_\theta(r)$.

4. Compute the pressure distribution, giving the result in the form $p(r) - p_i$, where $p_i$ is the known pressure on the surface of the inner cylinder.

5. Give the trajectory of the fluid particle located initially at $r = r_o$, $\theta = \theta_o$, and $z = z_o$. Show schematically the resulting path line.

6. Obtain the temperature distribution in the liquid $T(r)$ when the temperatures of the inner and outer cylinders are kept at constant values $T_i$ and $T_o$, respectively.
Problem 4: The solid cylinder of radius $a$ shown in the figure moves vertically with velocity $U$, dragging in its motion a coaxial cylindrical layer of liquid of constant uniform thickness $h$. The liquid has constant values of its density $\rho$, viscosity $\mu$, thermal conductivity $k$, and specific heat $c$. The air surrounding the liquid has a density $\rho_a \ll \rho$ and viscosity $\mu_a \ll \mu$, so that in the first approximation it moves without significant pressure differences, exerting on the liquid a negligible viscous force. Because of the symmetry of the problem, $\partial/\partial \theta = 0$ and the only nonzero velocity component is $v_z$.

1. Obtain the velocity $v_z(r, z)$ and pressure $p(r, z)$ in the liquid.

2. Compute the force on the cylinder (per unit length).

3. Calculate the volume flux $Q$ dragged by the cylinder, as well as the value of $U$ for which $Q = 0$.

4. Assuming that the thermal conductivity of air is very small compared with that of the liquid and that the cylinder is at constant uniform temperature $T_c$, obtain the temperature distribution in the liquid $T(r)$ as well as the conductive heat flux to the cylinder.